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**FINAL REPORT ON THE PROJECT
COMPUTATIONAL CONTROL
OF FLEXIBLE AEROSPACE SYSTEMS
(Grant No. NAG-1-1436)**

**Submitted to
NASA Langley Research Center**

**North Carolina A&T State University
Greensboro, North Carolina 27411**

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FINAL REPORT ON THE PROJECT
**COMPUTATIONAL CONTROL
OF FLEXIBLE AEROSPACE SYSTEMS**

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by

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ABSTRACT

The project, entitled "Computational Control of Flexible Aerospace Systems", was granted by NASA Langley Research Center (NAG-1-1436) started from January of 1992 and continued for three years. The main objective of this project is to establish a distributed parameter modeling technique for structural analysis, parameter estimation, vibration suppression and control synthesis of large flexible aerospace structures.

The major focus of the first-year (1992) research was the distributed parameter modeling of large flexible aerospace structures with complex configurations. As an example, the Low-power Atmospheric Compensation Experiment (LACE) Satellite Model had been used as the testbed and physical object. The main accomplishments can be summarized as follows. A physical research LACE Model with a 10 by 10 feet supporting frame had been built at NASA LaRC. The classical modal test had been conducted on this model so that the natural frequencies of the model were measured which were used to compare with the analytical results obtained from the distributed parameter model of the LACE Model. A comprehensive dynamic formulation of the distributed parameter model for tether-beam-rigid-body assembled structures by using transfer matrix method had been systematically derived. Although the mathematical model must be derived, depending on the feature of different satellite structures, the methodology developed in the first-year research is definitely suitable for all tether-beam-rigid-body assembled structures. The software PDEMOD were used to find the natural frequencies of the system. The analytical values were comparable to the experimental results. A technical report entitled "**Distributed Parameter Formulation of LACE Satellite Model by Using Transfer Matrix Method**" had been submitted to NASA LaRC^[1]. A summary of the report had been presented at the 9th VPI&SU Symposium on Dynamics & Control of Large Space Structures^[2]. The last two-year research was conducted on two phases. The first phase was the completion of the current version of PDEMOD Code and its related documentation. The second phase was to analyze the dynamic properties of a two-dimensional ground-based manipulator facility at the NASA Marshall Space Flight Center (MSFC) under various configurations, and to develop a methodology for vibration suppression of the end-effector by using distributed parameter modeling.

This report concentrates on the research outputs produced in the last two years. The main accomplishments can be summarized as follows. A new version of the PDEMOD Code had been completed based on several incomplete versions Mr. Taylor was working on just before he died. The verification of the code had been conducted by comparing the results with those examples for which the exact theoretical solutions can be obtained. A summary of the theoretical background of the package along with the verification examples has been reported in a technical paper submitted to the Joint Applied Mechanics & Material Conference, ASME, Los Angeles, June 28-30, 1995^[3]. Correspondingly, a brief USER'S MANUAL had been compiled, which mainly includes three parts: (1) Input data preparation, (2) Explanation of the Subroutines, (3) Specification of control variables.

Meanwhile, a theoretical investigation of the NASA MSFC two-dimensional ground-based manipulator facility by using distributed parameter modeling technique has been conducted. A new mathematical treatment for dynamic analysis and control of large flexible manipulator systems has been conceived, which may provide a embryonic form of a more sophisticated mathematical model for future modified versions of the PDEMOD Codes. This research has been reported in two technical papers^[4,5].

EXECUTIVE SUMMARY

The common approach used in the analysis of structural dynamics and the interaction with the control synthesis of large flexible aerospace structures is the finite element method. Although the finite element method has been widely accepted, the significant limitations still exist. The elements used in the finite element method are usually void of dynamics, such as massless axial springs. The consequence is that hundreds and thousands of elements are needed to represent large flexible structures in order to acquire analytical accuracy. Because of the computational cost and numerical inaccuracies involved in generating solutions of large number of equations, there is a practical limit to the accuracy of finite element dynamic models. The high order of the structural model requires an "order reduction" process before a control system can be designed. Seemingly unimportant modes can be inadvertently eliminated which prove later to be significant to control system performance and stability^[6].

Distributed parameter modeling is being seen to offer a viable alternative to the finite element approach for modeling large flexible space structures. Distributed parameter models have the advantages of improved accuracy, reduced number of modal parameters, the avoidance of modal order reduction, and especially, the ability to represent the structural and control system dynamics in the same system of equations. Continuum models have been made of several flexible space structures, which include the Spacecraft Control Laboratory (SCOLE)^[7], Solar Array Flight Experiment^[8], NASA Mini-Mast Truss^[9], the Space Station Freedom^[10], and recently, the Low-power Atmospheric Compensation Experiment (LACE) Satellite Model^[2]. A computer software package aiming at performing structural analysis and control system synthesis had been initialized and is primarily completed for its basic functions.

The software package PDEMOD was initialized by Dr. Lawrence W. Taylor, Jr. at NASA Langley Research Center during the middle of the 1980's. The first release of his work on PDEMOD package was in 1987^[11,12,13]. The initial interest of the package PDEMOD was to model the structural dynamics of general spacecraft configurations by using the distributed parameter approach. A system of partial differential equations is formulated and connected at their boundaries. The equations of motion for any number of rigid bodies are written in the frequency domain and in terms of the coefficients of the sinusoidal and hyperbolic functions which comprise the mode shapes. Distributed parameter models can, therefore, be generated for any three dimensional configurations describable by partial differential equations joined at their boundaries. The manual labor of generating such models is therefore avoided.

Because of Dr. Taylor's sudden passing away, it becomes a urgent task to summarize and sift his research achievements, and make it available to the other researchers. With the NASA's support, a new version of the PDEMOD Code has been completed during the past two years based on several incomplete versions of Dr. Taylor. A summary of the theoretical background of the package along with the verification examples has been reported in a technical paper^[3]. Summarily, a complex large aerospace structure is considered as an assembly of flexible beam elements and rigid bodies. Each flexible beam element is represented by four independent partial differential equations which exhibit lateral bending in two axes, axial deformation, and torsion. A system of partial differential equations is then formulated and connected at the elements' boundaries based on the compatibility conditions. The equations of motion for any number of rigid bodies are written in the frequency domain and in terms of the mode shape parameter coefficients. The deflections, forces and moments for both ends of a single beam element can be described in terms of the spatial derivatives of the solutions of the corresponding PDE's, further expressed in terms of the same set of mode shape parameter coefficients. Distributed parameter models can therefore be generated for any three-dimensional configurations describable by PDE's joined at their boundaries. An accomplishment of this task may provide an opportunity to more researchers to apply distributed parameter modeling techniques to a variety of aerospace structures.

The verification of the Code has been conducted by comparing the results with those examples for which the exact theoretical solutions can be obtained. Four verified examples were included in the package: Example 1 - Bending of a Cantilevered Beam; Example 2 - Bending of a Clamped-Clamped Beam; Example 3 - Bending of a Cantilevered Beam with a Tip-Mass M; Example 4 - Torsion of a Cantilevered Beam. Correspondingly, a brief USER'S MANUAL, had been compiled, which mainly includes three parts: (1) Input data preparation, (2) Explanation of the Subroutines, (3) Specification of control variables.

By investigating the potential of the distributed parameter modeling technique, Dr. Taylor and the other researchers expected that the PDEMOD may be further developed in the following aspects: (1) structural dynamics, modal frequencies and mode shapes; (2) parameter estimation of modal characteristics; (3) structural damping; (4) control system dynamics; and (5) design optimization. But, only the first of the program had been completed and included in the current package. To extend the functions of the current package, a massive research is being conducted, which suggests to modify the mathematical model and global system generating procedure^[5,9], to develop methodology for control synthesis^[4,14,15]. Instead of using the coefficients of the solution functions, the transfer matrix may be used finally, which provides a much more convenient way to describe the state-vector transition from one point of the structure to the other.

These tentative ideas have been included in the second-phase research. A theoretical investigation of the NASA MSFC two-dimensional ground-based manipulator facility by using distributed parameter modeling technique has been conducted. The MSFC facility is planned to conduct research in the berthing operation and, in general, research into the control of multibody configurations that are loosely coupled with flexible manipulator linkages^[16]. A new mathematical treatment for dynamic analysis and control of large flexible manipulator systems has been

conceived, which may provide a embryonic form of a more sophisticated mathematical model for future modified versions of the PDEMODO Codes. This research has been reported in two technical papers^[4,5].

ENCLOSURES

The enclosures of this report are listed as follows, which represent the research accomplishments of this project.

1. The PDEMODO Code and the computed results for four verified examples;
2. USER'S MANUAL;
3. Reference Paper 3: "PDEMODO - A Computer Program for Distributed Parameter Estimation of Flexible Aerospace Structures, Part I: Theory and Verification";
4. Reference Paper 4: "A Method of Superposing Rigid-Body Kinematics and Flexible Deflection for End-Effector Vibration Suppression of a Large Flexible Manipulator System".

SUGGESTION TO FUTURE RESEARCH

As mentioned before, the PDEMODO may be further developed in the following aspects: (1) structural dynamics, modal frequencies and mode shapes; (2) parameter estimation of modal characteristics; (3) structural damping; (4) control system dynamics; and (5) design optimization. To do so, the following two significant modifications will be necessary. First, the mathematical model supporting the current version of the PDEMODO Code must be modified. In current of the mode shape parameter coefficients of the corresponding beam elements where the rigid bodies were attached. In other words, the state variables were chosen as the coefficients of the sinusoidal and hyperbolic functions which comprise the solutions of the corresponding PDE's. This model is not suitable for control synthesis. Instead of using the coefficients of the solution functions, the transfer matrix may be used, which provides a much more straightforward way to describe the state-vector transition from one point of the structure to the other, directly using deflections, slopes, forces and moments at both ends of individual beam element as state variables. Since deflections are the controlled variables and forces and moments are the motivating variables, choosing them as state variables are definitely more suitable for control purpose.

Second, the package must include system identification and parameter estimation as an important part of the whole procedure. In general, the model used in distributed parameter analysis is actually an equivalent model of the real structure described by a set of PDE's. To keep the equivalency, a reasonable criterion to judge the equivalency must be properly set up first. The criterion used in current version of the PDEMODO was to keep the equivalency in the sense that the static stiffness were approximately equal between the distributed parameter model and the real structure. The equivalent parameters of the distributed parameter model, such as, mass, stiffness, radius of gyration, etc., were then determined based on this criterion. Further, the characteristic

equation is solved to obtain the dynamic properties of the structure. However, the stiffness of a complex structure is usually related to the frequencies. The static equivalency can not guarantee dynamic equivalency in general, and error may arise. To correct the errors, current PDEMODO package adjusted the equivalent parameters based on some experimentally measured frequencies without specified mathematical algorithm. This correction is largely dependent on the user's experience, different users obtain different results at times. To overcome this deficiency, it is necessary to provide an algorithm to precisely define the dynamic equivalency, such as maximum likelihood estimator.

ACKNOWLEDGEMENT

This work is funded by NASA Langley Research Center. Special thanks are due to Dr. Lawrence W. Taylor, Jr., the first-year technical monitor and the initiator of this project, and Dr. Claude R. Keckler, the current technical monitor. The authors would also like to acknowledge the help of Dr. Daniel P. Giesy and Mr. George A. Tan for familiarizing the computer system at NASA LaRC.

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ENCLOSURES

- 1. USER'S MANUAL**
- 2. The PDEMOD Code**
- 3. Reference Paper 3**
- 4. Reference Paper 4**

PDEMOD Program

USER'S MANUAL

PDEMOD

**A Computer Software Package
For Partial Differential Equation Modeling For Dynamic Analysis
of Large Flexible Aerospace Structures**

USER'S MANUAL

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INTRODUCTION

The software package **PDEMODO** was initialized by the late Lawrence W. Taylor, Jr. at NASA Langley Research Center during the middle of the 1980's. The first release of his work on **PDEMODO** package was in 1987. **PDEMODO** was initially developed to model the structural dynamics of general spacecraft configurations by using the distributed parameter approach, which consists of three-dimensional network of flexible beams and rigid bodies. The building blocks from which three-dimensional configurations can be constructed consist of (1) beams, which have bending in two directions, torsion, and elongation degrees of freedom, and (2) rigid bodies, which are connected by any network of beam elements. The full six degrees of freedom are allowed at either end of the beam. Rigid bodies can be attached to the beam at any angle or body location. The modified Bernoulli-Euler beam equation is used to represent the bending and the wave equations for torsion and elongation.

A system of partial differential equations (PDEs) is formulated and connected at the elements' boundaries based on the compatibility conditions. The equations of motion for any number of rigid bodies are written in the frequency domain and in terms of the coefficients of the sinusoidal and hyperbolic functions which comprise the mode shapes. The force and moment vectors for both ends of a single beam element can be described in terms of the spatial derivatives of the solutions of the corresponding PDE's. Distributed parameter models can therefore be generated for any three-dimensional configurations describable by PDEs joined at their boundaries. The manual labor of generating such models is therefore avoided.

Because of Dr. Taylor's sudden demise, it becomes an urgent task to summarize his research achievements and make them available to the other researchers. This work is being continued and this review may provide an opportunity to more researchers to apply distributed parameter modeling techniques to a variety of aerospace structures. The verification of the code has been conducted by comparing the results with those examples for which the exact theoretical solutions can be obtained, for instance, a simple beam with various boundary conditions, etc.

The theoretical derivation of the formulation and the verification of the code have been included in Shen, et al.^[1]. This **USER'S MANUAL** concentrates on the explanation of the package itself. The package **PDEMODO** mainly includes three parts: (1) Input data, which specifies structural configuration, mechanical properties of the consisting beams and rigid bodies, and the natural frequency range one would like to search, etc., (2) Main body of the package, which conducts the calculation specified by the formulation developed in Shen, et al.^[1], and (3) Subroutines necessary for completing the calculation. Since the formulation has been clearly described in Shen, et al.^[1], this Manual emphasizes the first and the third parts. It should please the user to know that it is necessary to read Shen, et al.^[1], before reading this Manual. The user should begin by solving some of the example problems given in Shen, et al.^[1]. The user should then proceed to work on their own complex problems.

The **USER'S MANUAL** and Shen, et al.^[1] are the basic technical specification reference documents for the **PDEMODO** package. Although these two reference documents are sufficient for some users, other references, e.g. [3-14], will give more details of this package. It is our recommendation that users review as many of these references as possible to gain a thorough understanding of the **PDEMODO** package.

Neither the late Lawrence W. Taylor, Jr., nor his working colleagues, the compilers of this package and the authors of some of the related papers, assume any responsibility for the validity, accuracy, or applicability of any results. Users must verify their own results.

ACKNOWLEDGMENTS

The software package **PDEMOD** was initialized by the late Lawrence W. Taylor, Jr. at NASA Langley Research Center. As an internationally recognized expert in system modeling, identification and control of aerospace and aeronautical structures, his great efforts and contributions to the initiation and development of the research on distributed parameter system modeling, identification and control will never be forgotten.

This research is sponsored and continuously supported by NASA Langley Research Center under Grant NAG-1-1436. The guidance and encouragement of the project directors, the late Lawrence W. Taylor, Jr., and Dr. Claude R. Keckler are greatly appreciated.

USER'S GUIDE

A. SUBROUTINES:

There are 17 subroutines in the package **PDEMOD**. The subroutines used in **PDEMOD** can be categorized as two types. The nine common-purpose subroutines (TYPE-I SUBROUTINES) are contained in a SUBROUTINE LIBRARY called "**LODLIB.F**", which is the selected part of the SUBROUTINE LIBRARY "**SYSPAC**" (SYStems analysis programs PACKage) at NASA Langley Research Center (LaRC)^[2]. The **SYSPAC** is a data base at LaRC for the purpose of making experimental data and a selection of analysis algorithms available to interested researchers studying aerodynamics, flight mechanics, structural dynamics and system identification. The other eight subroutines (TYPE-II SUBROUTINES) are not included in **SYSPAC** and are programmed specifically for **PDEMOD**. Their format is consistent with those of the TYPE-I SUBROUTINES.

All vectors and matrices used in the subroutines are expressed in a vector form, and have a common format. The first four elements of each vector are respectively: (1) the number of rows, (2) the number of columns, (3) the total number of elements, and (4) the data time interval. This format allows matrix information to be readily accessible in programming data analysis procedures, for calling numerous subroutines, in printing and in plotting. The 17 subroutines in **PDEMOD** are described as follows.

TYPE-I SUBROUTINES:

1. SUBROUTINE ADD (P, A, Q, B, C)

Description: Two compatible matrices (A and B) are multiplied by scalars (P and Q, respectively) and then added: $C=P*A+Q*B$.

2. SUBROUTINE MULT (A, B, C)

Description: Multiplies two matrices: $C=A*B$.

3. SUBROUTINE MAKE (A, B)

Description: Copies B in A: $A=B$.

4. SUBROUTINE SET (A, II, JJ)

Description: Creates a null matrix with II rows and JJ columns: $A=[0]$.

5. SUBROUTINE SPIT (A, B)

Description: Labels and lists a matrix.

6. SUBROUTINE TRANS (A, B)

Description: Generates a matrix transpose: $B=A^T$.

7. SUBROUTINE TILDA (A, B)

Description: Forms the matrix equivalent of a cross product from a vector $\{A\}_{3 \times 1}$,

$$B = \tilde{A} = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}$$

8. SUBROUTINE JUXTV (A, B, C)

Description: Combines by juxtaposition two compatible matrices in a vertical direction:

$$C = \begin{bmatrix} A \\ B \end{bmatrix}$$

9. SUBROUTINE IDENT (A, II)

Description: Forms an identity matrix A with dimension II.

TYPE-II SUBROUTINES:

10. SUBROUTINE AFORM (W, A, BODREF, CONFIG, L, P, Q, R, T, INRT, MASS, DUM, DUN, DUO, DUP)

Description: Forms the system matrix A. (See Shen, et al.^[1], Eq.34)

11. SUBROUTINE BODFORM (CONFIG, NBEAM, BODREF, NBODY)

Description: Determines rigid bodies' connectivity and the reference coordinate systems.

12. SUBROUTINE PFORM (W, L, Z, EIX, EIY, EAZ, EIP, MPL, IPL, FZ, AGKX, AGKY, PF, PM)

Description: Forms the matrices P_F and P_M (see Shen, et al.^[1], Eqs.16 and 17).

13. SUBROUTINE QFORM (W, L, Z, EIX, EIY, EAZ, EIP, MPL, IPL, FZ, AGKX, AGKY, QU, QS)

Description: Forms the matrices Q_u and Q_s (see Shen, et al.^[1], Eqs.14 and 15).

14. SUBROUTINE PQFORM (W, L, E, P, Q, DUM, DUN)

Description: Combines the matrices P and Q, respectively, by juxtaposition for multi-body, multi-beam system.

15. SUBROUTINE UPPER (A, DETA)

Description: Determines the determinant of the matrix A by transforming it as a upper triangular matrix.

16. SUBROUTINE DIAG (A, SHAPE)

Description: Diagrams the mode shapes.

17. SUBROUTINE WSEARCH (W, DW, A, DETNEW, DETOLD, WI)

Description: Search for the values of circular natural frequencies.

B. VARIABLES AND ARRAYS:

There are a number of variables and arrays are used in **PDEMOD**. They are defined below:

VARIABLES:

NBEAM: Number of beams.

NBODY: Number of rigid bodies.

NA=12*NBEAM, where the number 12 indicates that there are 12 unknown coefficients for each beam element (see Shen, et al.^[1], Eq.13).

Li: Length of the *i*th beam.

EIXi: Bending rigidity (EI_x) of the *i*th beam.

EIYi: Bending rigidity (EI_y) of the *i*th beam.

EIPi: Torsional rigidity (GI_p) of the *i*th beam.

EAZi: Axial rigidity (EA_z) of the *i*th beam.

MPLi: Mass per length of the *i*th beam.

IPLi: Mass moment of inertia per length of the *i*th beam.

FZi: Initial tensile force for the *i*th tether element.

AGKXi: Radius of gyration about x-axis of the cross-section area of the *i*th beam.

AGKYi: Radius of gyration about y-axis of the cross-section area of the *i*th beam.

MASSi: Mass of the *i*th rigid body.

W: Circular natural frequency.

WINC: Increment of the value of *W* for iteration.

NIW: Specified number of iteration.

ARRAYS:

CONFIG (NBEAM, 3): Denotes the structural configuration: Column No.1=Beam Identification Number (ID); Column No.2=Inboard Body Identification Number; Column No.3=Outboard Body Identification Number. A negative sign “-” prior to the numbers in columns 2 and 3 indicates which beam is used to define body axes. For example, a two-beam, three-body system is shown in Figure1. The definitions of “Inboard Body” and “Outboard Body” are depicted at the right-hand sides of the body ID Numbers. The 2 by 3 matrix **CONFIG** for this example is numbered as shown in the Table1.

Table 1 Example of the Matrix **CONFIG**

Beam's ID	Inboard Body's ID	Outboard Body's ID
1	-1	-2
2	2	-3

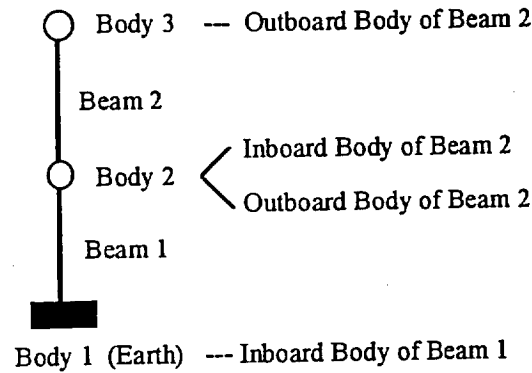


Figure 1 Two-Beam, Three-Body System

The negative signs prior to the numbers located at (1, 2) and (1, 3) indicate that both bodies 1 and 2 are defined in the beam 1's coordinate system, while the negative sign prior to the number located at (2, 3) indicates that body 3 is defined in the beam 2's coordinate system.

BODREF (NBODY, 2): Defines body's connectivity and reference coordinate. The number of rows of **BODREF** equals to the number of bodies. Each row provides the connectivity information of the corresponding body, consecutively. Column No.1 indicates the beam's ID, the connected body uses this beam's coordinate system as the reference system. Column No.2 indicates the mutual location between the body and the reference beam. If the body is the inboard body of that beam, the number equals to zero and if the body is the outboard body of that beam, the number equals to one. Note that **BODREF** is not a input array. All the numbers will be produced by calling **SUBROUTINE BODFORM**. In fact, **CONFIG** has provided all necessary information.

RiI (3, 3): The eccentric matrix at the attachment point between the *i*th beam and its inboard body.

RiO (3, 3): The eccentric matrix at the attachment point between the *i*th beam and its outboard body.

R (4+18*NBEAM): The eccentric matrix containing all of matrices by juxtaposition: **RiI** and **RiO**, *i*=1, NBEAM.

Ti (3, 3): The coordinate-transformation matrix.

INRTi (3, 3): The mass-moment-of-inertia matrix of the *i*th body.

INRT (4+9*NBODY): The mass-moment-of-inertia matrix containing all of **INRTi** matrices, *i*=1, NBODY, by juxtaposition.

E (4+9*NBEAM): Beams' mechanical-property matrix. The first four elements are used for the common purpose as mentioned before. Each consecutive nine numbers represent one beam's

mechanical properties, respectively, that is: $E(5)=EIX1$, $E(6)=EIY1$, $E(7)=EAZ1$, $E(8)=EIP1$, $E(9)=MPL1$, $E(10)=IPL1$, $E(11)=FZ1$, $E(12)=AGKX1$, $E(13)=AGKY1$, and so on.

MASS (4+NBODY): Body's mass matrix.

P [4+(3*12)*NBEAM]: Forms matrix **P** which consists of all the matrices P_F and P_M for each beam element consecutively by juxtaposition (see Shen, et al.[1], Eqs.16 and 17). Expressed in matrix form, the matrix **P** is constructed as

$$[P] = \begin{bmatrix} [P_{F_1}]_{3 \times 12} \\ [P_{M_1}]_{3 \times 12} \\ [P_{F_2}]_{3 \times 12} \\ [P_{M_2}]_{3 \times 12} \\ \vdots \\ \vdots \end{bmatrix} \begin{matrix} \text{For Beam1} \\ \\ \text{For Beam2} \\ \\ \vdots \end{matrix}$$

Q [4+(3*12)*NBEAM]: Forms matrix **Q** which consists of all the matrices Q_u and Q_s for each beam element consecutively by juxtaposition (see Shen, et al.[1], Eqs.14 and 15). Expressed in matrix form, the matrix **Q** is constructed as

$$[Q] = \begin{bmatrix} [Q_{u_1}]_{3 \times 12} \\ [Q_{s_1}]_{3 \times 12} \\ [Q_{u_2}]_{3 \times 12} \\ [Q_{s_2}]_{3 \times 12} \\ \vdots \\ \vdots \end{bmatrix} \begin{matrix} \text{For Beam1} \\ \\ \text{For Beam2} \\ \\ \vdots \end{matrix}$$

A [4+(12*NBEAM)*(12*NBEAM)]: Forms system matrix **A** which consists of all the matrices A_F and A_M for each beam element consecutively by juxtaposition, while the matrices A_F and A_M are constructed by the element matrices P_F , P_M , Q_u , and Q_s in the way indicated in Shen, et al.[1], Eq.33.

DETOLD, and DETNEW [4+(12*NBEAM)*(12*NBEAM)]: Determine the determinant of the system matrix **A** by iteration. **DETOLD** is the previous one, while **DETNEW** the up-dated one. When the value of the determinant is small enough to be considered as zero, the corresponding natural frequency is found.

C. CONTROL VARIABLES:

NPROB: The problem number the user chooses to solve. If **NPROB=1**, then **PROBLEM #1** is solved, and so forth. The current version of **PDEMOD** has included four verification examples:

EXAMPLE 1 - Bending of a Cantilevered Beam; EXAMPLE 2 - Bending of a Clamped-Clamped Beam; EXAMPLE 3 - Bending of a Cantilevered Beam with a Tip-Mass M; EXAMPLE 4 - Torsion of a Cantilevered Beam. User can add his own problems into the package and assign corresponding problem numbers.

IFREQ: Index for conducting natural frequency analysis. If **IFREQ=1**, the natural frequency of the system will be computed.

ISHP: Index for conducting mode shape analysis. If **ISHP=1**, the mode shape functions will be computed.

D. SPECIAL NUMBERS:

The following special numbers are used in the package to define some boundary conditions, null mass, or infinite mass, etc., so that the tedious modal reconstruction can be avoided.

1. For a null mass, or an imaginary mass, **MASSi**, at the free end of a cantilevered beam, the package defines **MASSi=0.9*10⁻⁸**. Correspondingly, the diagonal elements of its mass-moment-of-inertia matrix **INRTi** should be defined as the same number (**0.9*10⁻⁸**).
2. For infinite mass, such as, the mass of the foundation of a cantilevered beam, **MASSi**, the package defines **MASSi=99999999.0**. Correspondingly, the diagonal elements of its mass-moment-of-inertia matrix **INRTi** should be defined as the same number (**99999999.0**).
3. For restrictions of one-direction deflection, e.g., bending about x-axis, the package defines that the rigidity to resist the deflection in this direction approaches infinity, i.e., **EIXi=99999999.0**.
4. For the elements except tether element, the package defines the initial tensile force **FZi=0.0**.

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PDEMOD Program

Computer-Printout Codes

```

PROGRAM PDEMOD1
INTEGER BODREF,CONFIG,OBODY
REAL INRT1,INRT2,INRT3,INRT4,MASS1,MASS2,MASS3,MASS4,
1L1,L2,L3,MPL1,MPL2,MPL3,MASS,
2IPL1,IPL2,IPL3,L,INRT,INRTI,INTRO
DIMENSION DUM(1300), DUN(1300), DUO(1300),DUP(1300),
2INRT1(13),INRT2(13),INRT3(13),INRT4(13),
3R1I(13),R1O(13),R2I(13),R2O(13),R3I(13),R3O(13),
4E(40),A(2004),DETOLD(40),CONFIG(5,3),
5DETNEW(40),BW1(8),BW2(8),BW3(8),
7PF(40),PM(40),R(94),
8WI(1300),FI(1300),L(9),
9MASS(14),INRT(69),BW(34),TBODY(13),TBEAM(13),QU(40),QS(40)
DIMENSION RI(13),RO(13),INRTI(13),INRTO(13),P(436),Q(436),
1TIT(13),TRT(13),BODREF(10,2),R1(13),CGI(7),CGTOT(7),
2TI(13),SHAPE(2004),XJI(2004),D1(454),
3DELF(40),DELC(40),
4T(69),T1(13),T2(13),T3(13)
123 FORMAT(28H THE SYSTEM MATRIX WILL HAVE,I3,16HROWS AND COLUMNS)
543 FORMAT(4I5,E15.5)
707 FORMAT(7E14.5)
777 FORMAT(7I10)

C
C USER CAN CONTROL WHICH PROGRAM FUNCTIONS ARE ACTIVE(=1), AND NOT(=0)
IFREQ=1
ISHP=0

C
C USER MUST SELECT THE PROBLEM NUMBER DESIRED,i.e.NPROB=1,etc.
C PROBLEM #1 IS THE CANTILEVER BEAM BENDING;
C PROBLEM #2 IS THE CLAMPED-CLAMPED BEAM BENDING;
C PROBLEM #3 IS THE CANTILEVERED BEAM BENDING WITH A TIP_MASS;
C PROBLEM #4 IS THE CLAMPED-CLAMPED BEAM TORSION.
C
NPROB=2
if(nprob.eq.1) goto 1001
if(nprob.eq.2) goto 1002
if(nprob.eq.3) goto 1003
if(nprob.eq.4) goto 1004

C
C-PROBLEM 1: *****
C
1001 CONTINUE
C PROBLEM #1 is the CANTILEVERED BEAM BENDING (nbeam=1).
C THE MATRIX "CONFIG" DENOTES THE STRUCTURAL CONFIGURATION
C COL#1=BEAM I.D., COL#2-INBOARD BODY I.D., COL#3=OUTBOARD BODY
C NEGATIVE SIGN "-" DENOTES WHICH BEAM IS USED TO DEFINE BODY AXES.
c For the CANTILEVERED BEAM, BEAM1 links the Body1 (EARTH) to
c Body2 (a Null-mass). Body1 uses the Inboard end of Beam1 to
c define its axes. Body2 uses the outboard end of Beam1 to define
C its axes.
CONFIG(1,1)=1
CONFIG(1,2)=-1
CONFIG(1,3)=-2
NBEAM=1
DO 140 IBEAM = 1,NBEAM
140 PRINT 777, CONFIG(IBEAM,1),CONFIG(IBEAM,2),CONFIG(IBEAM,3)
NA=12*NBEAM
CALL BODFORM(CONFIG,NBEAM,BODREF,NBODY)
DO 163 I = 1,NBODY
163 PRINT 777,I,BODREF(I,1),BODREF(I,2)
CALL SET(WI,100,1)
WI(1)=0.
INPUT FOR BEAM#1, BODY#1 INBOARD AND BODY#2 OUTBOARD
L1=130.
EIX1=400000000.
EIY1=999999999.0

```

```

EIP1=999999999.0
MPL1=.09556
IPL1=.2907
EAZ1=999999999.0
FZ1=0.
AGKX1=999999999.
AGKY1=999999999.
CALL SET(R1I,3,1)
R1I(5)=0.
R1I(6)=0.
R1I(7)=0.
CALL TILDA(R1I,DUM)
CALL MAKE(R1I,DUM)
CALL SET(R1O,3,1)
R1O(5)=0.0
R1O(6)=0.0
R1O(7)=0.0
CALL TILDA(R1O,DUM)
CALL MAKE(R1O,DUM)
CALL SET(T1,3,3)
T1(5)=1.
T1(9)=1.
T1(13)=1.

```

C INPUT FOR BODY#1, THE FIXED END, THE EARTH.

```

MASS1=999999999.0
CALL SET(INRT1,3,3)
INRT1(5)=999999999.0
INRT1(9)=999999999.0
INRT1(13)=999999999.0
PRINT 707,MASS1

```

C INPUT FOR BODY#2, THE NULL-MASS.

```

MASS2=0.00000009
CALL SET(INRT2,3,3)
INRT2(5)=0.00000009
INRT2(9)=0.00000009
INRT2(13)=0.00000009
PRINT 707,MASS2
CALL SET(E,NBEAM,9)
E(5)=EIX1
E(6)=EIY1
E(7)=EAX1
E(8)=EIP1
E(9)=MPL1
E(10)=IPL1
E(11)=FZ1
E(12)=AGKX1
E(13)=AGKY1
CALL SET(L,NBEAM,1)
L(5)=L1
CALL SET(MASS,NBODY,1)
MASS(5)=MASS1
MASS(6)=MASS2
CALL MAKE(INRT,INRT1)
CALL JUXTV(INRT,INRT2,INRT)
CALL MAKE(T,T1)
CALL MAKE(R,R1I)
CALL JUXTV(R,R1O,R)
W=4.0
WINC=1.01
NIW=600
GO TO 1000

```

~
-PROBLEM 2: *****

C
1002 CONTINUE

C PROBLEM #2 is the CLAMPED_CLAMPED BEAM BENDING (nbeam=1).


```

C   THE MATRIX "CONFIG" DENOTES THE STRUCTURAL CONFIGURATION
C   COL#1=BEAM I.D., COL#2-INBOARD BODY I.D., COL#3=OUTBOARD BODY
C   NEGATIVE SIGN "-" DENOTES WHICH BEAM IS USED TO DEFINE BODY AXES.
C   For the CANTILEVERED BEAM, BEAM1 links the Body1 (EARTH) to
C   Body2(EARTH,EITHER). Body1 uses the Inboard end of Beam1 to
~   define its axes. Body2 uses the outboard end of Beam1 to define
    its axes.
    CONFIG(1,1)=1
    CONFIG(1,2)=-1
    CONFIG(1,3)=-2
    NBEAM=1
    DO 240 IBEAM = 1,NBEAM
240  PRINT 777, CONFIG(IBEAM,1),CONFIG(IBEAM,2),CONFIG(IBEAM,3)
    NA=12*NBEAM
    CALL BODFORM(CONFIG,NBEAM,BODREF,NBODY)
    DO 263 I = 1,NBODY
263  PRINT 777,I,BODREF(I,1),BODREF(I,2)
    CALL SET(WI,100,1)
    WI(1)=0.
C   INPUT FOR BEAM#1, BODY#1 INBOARD AND BODY#2 OUTBOARD
    L1=130.
    EIX1=40000000.
    EIY1=999999999.0
    EIP1=999999999.0
    MPL1=.09556
    IPL1=.2907
    EAZ1=999999999.0
    FZ1=0.
    AGKX1=9999.
    AGKY1=999999999.
    CALL SET(R1I,3,1)
    R1I(5)=0.
    R1I(6)=0.
    R1I(7)=0.
    CALL TILDA(R1I,DUM)
    CALL MAKE(R1I,DUM)
    CALL SET(R1O,3,1)
    R1O(5)=0.0
    R1O(6)=0.0
    R1O(7)=0.0
    CALL TILDA(R1O,DUM)
    CALL MAKE(R1O,DUM)
    CALL SET(T1,3,3)
    T1(5)=1.
    T1(9)=1.
    T1(13)=1.
C   INPUT FOR BODY#1, THE FIXED END, THE EARTH.
    MASS1=999999999.0
    CALL SET(INRT1,3,3)
    INRT1(5)=9999999.0
    INRT1(9)=9999999.0
    INRT1(13)=9999999.0
    PRINT 707,MASS1
C   INPUT FOR BODY#2, THE NULL-MASS.
    MASS2=999999999.0
    CALL SET(INRT2,3,3)
    INRT2(5)=9999999.0
    INRT2(9)=9999999.0
    INRT2(13)=9999999.0
    PRINT 707,MASS2
    CALL SET(E,NBEAM,9)
    E(5)=EIX1
    E(6)=EIY1
    E(7)=EAZ1
    E(8)=EIP1
    E(9)=MPL1

```

```

E(10)=IPL1
E(11)=FZ1
E(12)=AGKX1
E(13)=AGKY1
CALL SET(L,NBEAM,1)
L(5)=L1
CALL SET(MASS,NBODY,1)
MASS(5)=MASS1
MASS(6)=MASS2
CALL MAKE(INRT,INRT1)
CALL JUXTV(INRT,INRT2,INRT)
CALL MAKE(T,T1)
CALL MAKE(R,R1I)
CALL JUXTV(R,R1O,R)
W=25.0
WINC=1.01
NIW=300
GO TO 1000

```

```

C
C-PROBLEM 3 *****

```

```

C
1003 CONTINUE
C PROBLEM #3 is the CANTILEVERED BEAM BENDING (nbeam=1) with a
C TIP-BODY CONNECTED.
C THE MATRIX "CONFIG" DENOTES THE STRUCTURAL CONFIGURATION
C COL#1=BEAM I.D., COL#2-INBOARD BODY I.D., COL#3=OUTBOARD BODY
C NEGATIVE SIGN "-" DENOTES WHICH BEAM IS USED TO DEFINE BODY AXES.
C For the CANTILEVERED BEAM, BEAM1 links the Body1 (EARTH) to
C Body2. Body1 uses the Inboard end of Beam1 to
C define its axes. Body2 uses the outboard end of Beam1 to define
C its axes.
CONFIG(1,1)=1
CONFIG(1,2)=-1
CONFIG(1,3)=-2
NBEAM=1
DO 340 IBEAM = 1,NBEAM
340 PRINT 777, CONFIG(IBEAM,1),CONFIG(IBEAM,2),CONFIG(IBEAM,3)
NA=12*NBEAM
CALL BODFORM(CONFIG,NBEAM,BODREF,NBODY)
DO 363 I = 1,NBODY
363 PRINT 777,I,BODREF(I,1),BODREF(I,2)
CALL SET(WI,100,1)
WI(1)=0.
C INPUT FOR BEAM#1, BODY#1 INBOARD AND BODY#2 OUTBOARD
L1=3.077
EIX1=175.9644
EIY1=999999999.0
EIP1=999999999.0
MPL1=0.012
IPL1=1.0e10
EAZ1=999999999.0
FZ1=0.
AGKX1=999999999.
AGKY1=999999999.
CALL SET(R1I,3,1)
R1I(5)=0.
R1I(6)=0.
R1I(7)=0.
CALL TILDA(R1I,DUM)
CALL MAKE(R1I,DUM)
CALL SET(R1O,3,1)
R1O(5)=0.0
R1O(6)=0.0
R1O(7)=-0.07
CALL TILDA(R1O,DUM)
CALL MAKE(R1O,DUM)

```

```

CALL SET(T1,3,3)
T1(5)=1.
T1(9)=1.
T1(13)=1.
C INPUT FOR BODY#1, THE CLAMPED_END, THE EARTH.
MASS1=999999999.0
CALL SET(INRT1,3,3)
INRT1(5)=999999999.0
INRT1(9)=999999999.0
INRT1(13)=999999999.0
PRINT 707,MASS1
C INPUT FOR BODY#2, THE REFLECTOR.
MASS2=4.952/32.2
CALL SET(INRT2,3,3)
INRT2(5)=2.341e-4
INRT2(9)=2.341e-4
INRT2(13)=1.524e-3
PRINT 707,MASS2
CALL SET(E,NBEAM,9)
E(5)=EIX1
E(6)=EIIY1
E(7)=EAI1
E(8)=EIP1
E(9)=MPL1
E(10)=IPL1
E(11)=FZ1
E(12)=AGKX1
E(13)=AGKY1
CALL SET(L,NBEAM,1)
L(5)=L1
CALL SET(MASS,NBODY,1)
MASS(5)=MASS1
MASS(6)=MASS2
CALL MAKE(INRT,INRT1)
CALL JUXTV(INRT,INRT2,INRT)
CALL MAKE(T,T1)
CALL MAKE(R,R1I)
CALL JUXTV(R,R1O,R)
W=10.0
winc=1.01
niw=150
GO TO 1000

C
C-PROBLEM 4 *****
C
1004 CONTINUE
C PROBLEM #4 is the TORSION OF A CLAMOED-CLAMPED BEAM (nbeam=1)
C WITHOUT BODY CONNECTED.
C THE MATRIX "CONFIG" DENOTES THE STRUCTURAL CONFIGURATION
C COL#1=BEAM I.D., COL#2-INBOARD BODY I.D., COL#3=OUTBOARD BODY
C NEGATIVE SIGN "-" DENOTES WHICH BEAM IS USED TO DEFINE BODY AXES.
C For the CANTILEVERED BEAM, BEAM1 links the Body1 (EARTH) to
C Body2(another clamped end). Body1 uses the Inboard end of Beam1 to
C define its axes. Body2 uses the outboard end of Beam1 to define
C its axes.
CONFIG(1,1)=1
CONFIG(1,2)=-1
CONFIG(1,3)=-2
NBEAM=1
DO 440 IBEAM = 1,NBEAM
440 PRINT 777,CONFIG(IBEAM,1),CONFIG(IBEAM,2),CONFIG(IBEAM,3)
NA=12*NBEAM
CALL BODFORM(CONFIG,NBEAM,BODREF,NBODY)
do 463 i=1,nbody
463 print 777, i,bodref(i,1),bodref(i,2)
CALL SET(WI,100,1)

```

```

C      WI(1)=0.
      INPUT FOR BEAM 1, BODY#1 INBOARD, BODY#2 OUTBOARD
      L1=130.
      EIX1=999999999.0
      EIY1=999999999.0
      EIP1=400000000.0
      MPL1=0.09556
      IPL1=.2907
      EAZ1=999999999.0
      FZ1=0.
      AGKX1=999999999.0
      AGKY1=999999999.0
      CALL SET(R1I,3,1)
      R1I(5)=0.
      R1I(6)=0.
      R1I(7)=0.
      CALL TILDA(R1I,DUM)
      CALL MAKE(R1I,DUM)
      CALL SET(R1O,3,1)
      R1O(5)=0.
      R1O(6)=0.
      R1O(7)=0.
      CALL TILDA(R1O,DUM)
      CALL MAKE(R1O,DUM)
      CALL SET(T1,3,3)
      T1(5)=1.
      T1(9)=1.
      T1(13)=1.
C      INPUT FOR BODY 1(one clamped end)
      MASS1=999999999.
      CALL SET(INRT1,3,3)
      INRT1(5)=999999999.0
      INRT1(9)=999999999.0
      INRT1(13)=999999999.0
      PRINT 707, MASS1
C      INPUT FOR BODY 2 (another clamped end)
      MASS2=999999999.0
      CALL SET(INRT2,3,3)
      INRT2(5)=999999999.0
      INRT2(9)=999999999.0
      INRT2(13)=999999999.0
      PRINT 707, MASS2
      CALL SET(E,NBEAM,9)
      E(5)=EIX1
      E(6)=EIY1
      E(7)=EAZ1
      E(8)=EIP1
      E(9)=MPL1
      E(10)=IPL1
      E(11)=FZ1
      E(12)=AGKX1
      E(13)=AGKY1
      CALL SET(L,NBEAM,1)
      L(5)=L1
      CALL SET(MASS,NBODY,1)
      MASS(5)=MASS1
      MASS(6)=MASS2
      CALL MAKE(INRT,INRT1)
      CALL JUXTV(INRT,INRT2,INRT)
      CALL MAKE(T,T1)
      CALL MAKE(R,R1I)
      CALL JUXTV(R,R1O,R)
      W=280.0
      WINC=1.01
      NIW=300
      GO TO 1000

```

```

C          ***** THE COMMON PART OF THE MAIN PROGRAM *****
C
C
1000  CALL SPIT(L,2H L)
      CALL SPIT(E,2H E)
      CALL SPIT(R,2H R)
      CALL SPIT(T,2H T)
      CALL SPIT(MASS,5H MASS)
      CALL SPIT(INRT,5H INRT)
      CALL SET(DETNEW,NA,1)
      DO 1 IW=1,NIW
      DW=W*(WINC-1.)
      W=W+DW
C      FORM FORCE AND MOMENT MATRICES "PF" AND "PM"
C      FORM LINEAR AND ANGULAR DEFLECTION MATRICES "QU" AND "QS"
      CALL PQFORM(W,L,E,P,Q,DUM,DUN)
      CALL SET(A,NA,NA)
      CALL AFORM(W,A,BODREF,CONFIG,L,P,Q,R,T,INRT,MASS,
1DUM,DUN,DUO,DUP)
      CALL ADD(1.,A,0.,A,A)
      CALL WSEARCH(W,DW,A,DETNEW,DETOLD,WI)
1      CONTINUE
      AD=1./6.283185
      CALL ADD(AD,WI,0.,WI,FI)
      CALL SPIT(FI,3H FI)
      IF(ISHP)998,998,31
31      CONTINUE
      NW=WI(1)+.01
      DO 3 IW=1,NW
      W=WI(IW+4)
      CALL PQFORM(W,L,E,P,Q,DUM,DUN)
      CALL AFORM(W,A,BODREF,CONFIG,L,P,Q,R,T,INRT,MASS,
1DUM,DUN,DUO,DUP)
      CALL UPPER(A,DETNEW)
      DETN=1.
      DO 92 I = 1,NA
92      DETN=DETN*DETNEW(I+4)
      CALL SET(DUM,1,NA)
      DUM(NA+4)=1.
      CALL DIAG(A,DUM)
      IF(IW-1)29,30,29
30      CALL MAKE(SHAPE,DUM)
      GO TO 28
29      CALL JUXTV(SHAPE,DUM,SHAPE)
28      CONTINUE
3      CONTINUE
      IF(ISHP)998,998,999
999      CALL SPIT(SHAPE,5H SHAPE)
998      STOP
      END

```

```

C          *****          *****          *****
C          SUBROUTINES
C          *****          *****          *****
      SUBROUTINE AFORM(W,A,BODREF,CONFIG,L,P,Q,R,T,INRT,MASS,
1DUM,DUN,DUO,DUP)
      INTEGER APPEND,BEAM1,BODREF,CONFIG,OBODY
      REAL INRT,INRTI,INRTO,MASS,L
      DIMENSION DUM(1300),DUN(1300),DUO(1300),DUP(1300),
2INRT(69),INRTI(13),MASS(14),L(9),R(94),RI(13),R1(13),
3A(2004),P(436),PF(40),PM(40),Q(436),QU(40),QS(40),
4BODREF(10,2),CONFIG(5,3),T(49),TBEAM(13),T1(13),
5TIT(13),TRT(13),QU1(40),QS1(40),TI(13),RO(13),
6INRTO(13)
707      FORMAT(7E15.4)
      NBODY=MASS(1)+.01

```

```

NBEAM=L(1)+.01
NA=NBEAM*12
APPEND=NBODY
DO 2 IBEAM = 1,NBEAM
CALL SET(TBEAM,3,3)
CALL SET(RI,3,3)
CALL SET(RO,3,3)
DO 13 IBLOCK = 1,9
TBEAM(IBLOCK+4)=T((IBEAM*3-3)*3+IBLOCK+4)
RI(IBLOCK+4)=R((IBEAM*6-6)*3+IBLOCK+4)
RO(IBLOCK+4)=R((IBEAM*6-3)*3+IBLOCK+4)
FORM MATRIX "A" FOR INBOARD BODY
CALL SET(QU,3,12)
CALL SET(QS,3,12)
DO 12 IBLOCK = 1,36
QU(IBLOCK+4)=Q((IBEAM*12-12)*12+IBLOCK+4)
QS(IBLOCK+4)=Q((IBEAM*12-9)*12+IBLOCK+4)
CALL TRANS(TBEAM,TIT)
IBODY=CONFIG(IBEAM,2)
IF(IBODY)6,4,4
IBODY=-IBODY
ENREF=1.
GO TO 18
ENREF=0.
BEAM1=BODREF(IBODY,1)
IO=BODREF(IBODY,2)
IF(BEAM1-IBEAM)70,71,70
CALL SET(T1,3,3)
CALL SET(QU1,3,12)
CALL SET(QS1,3,12)
CALL SET(R1,3,3)
DO 72 IBLOCK = 1,9
T1(IBLOCK+4)=T((BEAM1*3-3)*3+IBLOCK+4)
R1(IBLOCK+4)=R((BEAM1*6+3*IO-6)*3+IBLOCK+4)
DO 73 IBLOCK = 1,36
QU1(IBLOCK+4)=Q((BEAM1*12+6*IO-12)*12+IBLOCK+4)
QS1(IBLOCK+4)=Q((BEAM1*12+6*IO-9)*12+IBLOCK+4)
CALL MULT(T1,QU1,DUN)
CALL ADD(1.,RI,-1.,R1,DUM)
CALL MULT(DUM,T1,DUO)
CALL MULT(DUO,QS1,DUM)
CALL ADD(-1.,DUN,-1.,DUM,DUO)
CALL MULT(TIT,DUO,DUP)
CALL MULT(T1,QS1,DUM)
CALL MULT(TIT,DUM,DUN)
CALL ADD(-1.,DUN,0.,DUN,DUN)
CALL JUXTV(DUP,DUN,DUO)
CALL JUXTV(QU,QS,DUP)
DO 75 IBLOCK = 1,6
DO 74 JBLOCK = 1,12
ID=(APPEND*6+IBLOCK-1)*NA+JBLOCK+BEAM1*12-8
A(ID)=DUO((IBLOCK-1)*12+JBLOCK+4)
DO 75 JBLOCK = 1,12
ID=(APPEND*6+IBLOCK-1)*NA+JBLOCK+IBEAM*12-8
A(ID)=DUP((IBLOCK-1)*12+JBLOCK+4)
APPEND=APPEND+1
CONTINUE
CONTINUE
EM=MASS(IBODY+4)
CALL SET(INRTI,3,3)
DO 14 IBLOCK = 1,9
INRTI(IBLOCK+4)=INRT((IBODY*3-3)*3+IBLOCK+4)
CALL SET(PF,3,12)
CALL SET(PM,3,12)
DO 31 IBLOCK = 1,36
PF(IBLOCK+4)=P((IBEAM*12-12)*12+IBLOCK+4)

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31 PM(IBLOCK+4)=P((IBEAM*12-9)*12+IBLOCK+4)
W2=W*W
W2IN=1./W2
CALL MULT(RI,TBEAM,DUM)
CALL MULT(TIT,DUM,TRT)
FIRST BLOCK, FORCE EQUATIONS, INBOARD
IF(EM-999999999.)86,87,86
87 AD=ENREF
AE=0.
GO TO 88
86 AD=ENREF*EM
AE=W2IN
88 CONTINUE
CALL ADD(AD,QU,AE,PF,DUO)
CALL MULT(TRT,QS,DUN)
CALL ADD(1.,DUO,+AD,DUN,DUO)
C SECOND BLOCK, MOMENT EQUATIONS, INBOARD
CALL MULT(RI,TBEAM,DUM)
CALL MULT(DUM,PF,DUN)
CALL MULT(TBEAM,PM,DUM)
CALL ADD(+W2IN,DUM,+W2IN,DUN,DUP)
CALL MULT(TBEAM,QS,DUM)
CALL MULT(INRTI,DUM,DUN)
CALL ADD(1.,DUP,ENREF,DUN,DUP)
CALL JUXTV(DUO,DUP,DUO)
DO 5 IBLOCK = 1,6
DO 5 JBLOCK = 1,12
5 A((IBODY*6+IBLOCK-7)*NA+IBEAM*12+JBLOCK-8)=DUO((IBLOCK-1)
1*12+JBLOCK+4)
C FORM MATRIX "A" FOR OUTBOARD BODY
CALL SET(QU,3,12)
CALL SET(QS,3,12)
DO 19 IBLOCK = 1,36
QU(IBLOCK+4)=Q((IBEAM*12-6)*12+IBLOCK+4)
QS(IBLOCK+4)=Q((IBEAM*12-3)*12+IBLOCK+4)
OBODY=CONFIG(IBEAM,3)
IF(OBODY)15,15,16
15 ENREF=1.
OBODY=-OBODY
GO TO 17
16 ENREF=0.
BEAM1=BODREF(OBODY,1)
IO=BODREF(OBODY,2)
IF(BEAM1-IBEAM)80,81,80
80 CALL SET(T1,3,3)
CALL SET(QU1,3,12)
CALL SET(QS1,3,12)
CALL SET(R1,3,3)
DO 82 IBLOCK = 1,9
T1(IBLOCK+4)=T((BEAM1*3-3)*3+IBLOCK+4)
82 R1(IBLOCK+4)=R((BEAM1*3-3)*3+IBLOCK+4)
DO 83 IBLOCK = 1,36
QU1(IBLOCK+4)=Q((BEAM1*12+6*IO-12)*12+IBLOCK+4)
83 QS1(IBLOCK+4)=Q((BEAM1*12+6*IO-9)*12+IBLOCK+4)
CALL MULT(T1,QU1,DUN)
CALL ADD(1.,RI,-1.,R1,DUM)
CALL MULT(DUM,T1,DUO)
CALL MULT(DUO,QS1,DUM)
CALL ADD(-1.,DUN,-1.,DUM,DUO)
CALL MULT(TIT,DUO,DUP)
CALL MULT(T1,QS1,DUM)
CALL MULT(TIT,DUM,DUN)
CALL ADD(-1.,DUN,0.,DUN,DUN)
CALL JUXTV(DUP,DUN,DUO)
555 CALL JUXTV(QU,QS,DUP)
DO 85 IBLOCK = 1,6

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```

DO 84 JBLOCK = 1,12
ID=(APPEND*6+IBLOCK-1)*NA+JBLOCK+BEAM1*12-8
84 A(ID)=DUO((IBLOCK-1)*12+JBLOCK+4)
DO 85 JBLOCK = 1,12
ID=(APPEND*6+IBLOCK-1)*NA+JBLOCK+IBEAM*12-8
~5 A(ID)=DUP((IBLOCK-1)*12+JBLOCK+4)
APPEND=APPEND+1
81 CONTINUE
17 CONTINUE
EM=MASS(OBODY+4)
CALL SET(INRTO,3,3)
DO 24 IBLOCK = 1,9
24 INRTO(IBLOCK+4)=INRT((OBODY*3-3)*3+IBLOCK+4)
CALL SET(PF,3,12)
CALL SET(PM,3,12)
DO 33 IBLOCK = 1,36
33 PF(IBLOCK+4)=P((IBEAM*12-6)*12+IBLOCK+4)
PM(IBLOCK+4)=P((IBEAM*12-3)*12+IBLOCK+4)
W2=W*W
W2IN=1./W2
CALL MULT(RO,TBEAM,DUM)
CALL MULT(TIT,DUM,TRT)
C FIRST BLOCK, FORCE EQUATIONS, OUTBOARD
IF(EM-999999999.)96,97,96
97 AD=ENREF
AE=0.
GO TO 98
96 AD=ENREF*EM
AE=W2IN
98 CONTINUE
CALL ADD(AD,QU,AE,PF,DUO)
CALL MULT(TRT,QS,DUN)
CALL ADD(1.,DUO,+AD,DUN,DUO)
SECOND BLOCK, MOMENT EQUATIONS, OUTBOARD
CALL MULT(RO,TBEAM,DUM)
CALL MULT(DUM,PF,DUN)
CALL MULT(TBEAM,PM,DUM)
CALL ADD(+W2IN,DUM,+W2IN,DUN,DUP)
CALL MULT(TBEAM,QS,DUM)
CALL MULT(INRTO,DUM,DUN)
CALL ADD(1.,DUP,ENREF,DUN,DUP)
CALL JUXTV(DUO,DUP,DUO)
DO 7 IBLOCK = 1,6
DO 7 JBLOCK = 1,12
7 A((OBODY*6+IBLOCK-7)*NA+IBEAM*12+JBLOCK-8)=DUO((IBLOCK-1)
1*12+JBLOCK+4)
2 CONTINUE
RETURN
END
C
SUBROUTINE BODFORM(CONFIG,NBEAM,BODREF,NBODY)
INTEGER CONFIG,BODREF
DIMENSION CONFIG(5,3),BODREF(10,2)
777 FORMAT(7I10)
JMAX=1
DO 50 IBEAM = 1,NBEAM
J=CONFIG(IBEAM,2)
J2=J*J
IF(JMAX*JMAX-J2)1,2,2
1 JMAX2=J2
2 CONTINUE
IF(J)51,51,52
J=-J
BODREF(J,1)=IBEAM
BODREF(J,2)=0
52 J=CONFIG(IBEAM,3)

```



```

J2=J*J
IF (JMAX*JMAX-J2) 3, 4, 4
3 JMAX2=J2
4 CONTINUE
IF (J) 53, 53, 54
53 J=-J
BODREF (J, 1)=IBEAM
BODREF (J, 2)=1
54 CONTINUE
50 CONTINUE
AD=JMAX2
NBODY=SQRT(AD)+.01
DO 63 I = 1, NBODY
63 PRINT 777, I, BODREF (I, 1), BODREF (I, 2)
RETURN
END

C
SUBROUTINE PFORM(W, L, Z, EIX, EIY, EAZ, EIP, MPL, IPL, FZ, AGKX, AGKY,
1 PF, PM)
REAL L, MPL, IPL
DIMENSION PF(40), PM(40)
707 FORMAT(7E14.5)
W2=W*W
W2IN=1./W2
IF (EIX-FZ*FZ*.071) 8, 8, 9
8 ARG=MPL/(FZ-EIX*MPL*W2/AGKX)
BXAB=W*SQRT(ARG)
BXCD=18.42/L
GO TO 10
9 CONTINUE
AD=.5*(MPL*W2/AGKX-FZ/EIX)
AE=MPL*W2/EIX
ARG=AD+SQRT(AD*AD+AE)
BXAB=SQRT(ARG)
ARG=-AD+SQRT(AD*AD+AE)
BXCD=SQRT(ARG)
10 CONTINUE
IF (EIY-FZ*FZ*.071) 11, 11, 12
11 ARG=MPL/(FZ-EIY*MPL*W2/AGKY)
BYAB=W*SQRT(ARG)
BYCD=18.42/L
GO TO 13
12 CONTINUE
AD=.5*(MPL*W2/AGKY-FZ/EIY)
AE=MPL*W2/EIY
ARG=AD+SQRT(AD*AD+AE)
BYAB=SQRT(ARG)
ARG=-AD+SQRT(AD*AD+AE)
BYCD=SQRT(ARG)
13 CONTINUE
BZ=SQRT(MPL/EAZ)*W
BP=SQRT(IPL/EIP)*W
C FORCE MATRIX FOR BEAM
CALL SET(PF, 3, 12)
BXAB2=BXAB*BXAB
BXCD2=BXCD*BXCD
BXAB3=BXAB2*BXAB
BXCD3=BXCD2*BXCD
BYAB2=BYAB*BYAB
BYCD2=BYCD*BYCD
BYAB3=BYAB2*BYAB
BYCD3=BYCD2*BYCD
IF (Z) 2, 3, 2
3 PF(5)=+BXAB3*EIX+BXAB*FZ
PF(6)=0.
PF(7)=-BXCD3*EIX-BXCD*FZ

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PF(8)=0.
PF(21)=+BYAB3*EIY+BYAB*FZ
PF(22)=0.
PF(23)=-BYCD3*EIY-BYCD*FZ
PF(24)=0.
PF(37)=BZ*EAZ
PF(38)=0.
GO TO 4
2 PF(5)=-BXAB3*EIX*COS(BXAB*L)-BXAB*FZ*COS(BXAB*L)
PF(6)=BXAB3*EIX*SIN(BXAB*L)+BXAB*FZ*SIN(BXAB*L)
PF(7)=BXCD3*EIX*COSH(BXCD*L)-BXCD*FZ*COSH(BXCD*L)
PF(8)=BXCD3*EIX*sinh(BXCD*L)-BXCD*FZ*sinh(BXCD*L)
PF(21)=-BYAB3*EIY*cos(BYAB*L)-BYAB*FZ*cos(BYAB*L)
PF(22)=BYAB3*EIY*sin(BYAB*L)+BYAB*FZ*sin(BYAB*L)
PF(23)=BYCD3*EIY*cosh(BYCD*L)-BYCD*FZ*cosh(BYCD*L)
PF(24)=BYCD3*EIY*sinh(BYCD*L)-BYCD*FZ*sinh(BYCD*L)
PF(37)=-BZ*EAZ*cos(BZ*L)
PF(38)=BZ*EAZ*sin(BZ*L)
4 CONTINUE
C MOMENT MATRIX FOR BEAM
CALL SET(PM,3,12)
IF(Z)5,6,5
6 PM(17)=0.
PM(18)=+BXAB2*EIX+FZ
PM(19)=0.
PM(20)=-BXCD2*EIX+FZ
PM(9)=0.
PM(10)=-BYAB2*EIY-FZ
PM(11)=0.
PM(12)=+BYCD2*EIY-FZ
PM(39)=BP*EIP
PM(40)=0.
GO TO 7
PM(17)=-BXAB2*EIX*SIN(BXAB*L)-FZ*SIN(BXAB*L)
PM(18)=-BXAB2*EIX*COS(BXAB*L)-FZ*COS(BXAB*L)
PM(19)=BXCD2*EIX*sinh(BXCD*L)-FZ*sinh(BXCD*L)
PM(20)=BXCD2*EIX*cosh(BXCD*L)-FZ*cosh(BXCD*L)
PM(9)=BYAB2*EIY*sin(BYAB*L)+FZ*sin(BYAB*L)
PM(10)=BYAB2*EIY*cos(BYAB*L)+FZ*cos(BYAB*L)
PM(11)=-BYCD2*EIY*sinh(BYCD*L)+FZ*sinh(BYCD*L)
PM(12)=-BYCD2*EIY*cosh(BYCD*L)+FZ*cosh(BYCD*L)
PM(39)=-BP*EIP*cos(BP*L)
PM(40)=BP*EIP*sin(BP*L)
7 CONTINUE
RETURN
END
C SUBROUTINE QFORM(W,L,Z,EIX,EIY,EAZ,EIP,MPL,IPL,FZ,AGKX,AGKY,QU,QS)
REAL L,MPL,IPL
DIMENSION QU(40),QS(40)
W2=W*W
IF(EIX-FZ*FZ*.071)1,1,2
1 ARG=MPL/(FZ-EIX*MPL*W2/AGKX)
BXAB=W*SQRT(ARG)
BXCD=18.42/L
GO TO 3
2 CONTINUE
AD=.5*(MPL*W2/AGKX-FZ/EIX)
AE=MPL*W2/EIX
ARG=AD+SQRT(AD*AD+AE)
BXAB=SQRT(ARG)
ARG=-AD+SQRT(AD*AD+AE)
BXCD=SQRT(ARG)
3 CONTINUE
IF(EIY-FZ*FZ*.071)4,4,5
4 ARG=MPL/(FZ-EIY*MPL*W2/AGKY)

```

```

BYAB=W*SQRT(ARG)
BYCD=18.42/L
GO TO 6
5 CONTINUE
AD=.5*(MPL*W2/AGKY-FZ/EIY)
AE=MPL*W2/EIY
ARG=AD+SQRT(AD*AD+AE)
BYAB=SQRT(ARG)
ARG=-AD+SQRT(AD*AD+AE)
BYCD=SQRT(ARG)
6 CONTINUE
BZ=SQRT(MPL/EAZ)*W
BP=SQRT(IPL/EIP)*W
AD=1.
C LINEAR DEFLECTION MATRIX
CALL SET(QU,3,12)
IF(Z)8,9,8
9 QU(5)=0.
QU(6)=1.
QU(7)=0.
QU(8)=1.
QU(21)=0.
QU(22)=1.
QU(23)=0.
QU(24)=1.
QU(37)=0.
QU(38)=1.
GO TO 10
8 CONTINUE
QU(5)=SIN(BXAB*L)
QU(6)=COS(BXAB*L)
QU(7)=SINH(BXCD*L)
QU(8)=COSH(BXCD*L)
QU(21)=SIN(BYAB*L)
QU(22)=COS(BYAB*L)
QU(23)=SINH(BYCD*L)
QU(24)=COSH(BYCD*L)
QU(37)=SIN(BZ*L)
QU(38)=COS(BZ*L)
10 CALL ADD(AD,QU,0.,QU,QU)
C ANGULAR DEFLECTION MATRIX
CALL SET(QS,3,12)
IF(Z)11,12,11
12 QS(17)=BXAB
QS(18)=0.
QS(19)=BXCD
QS(20)=0.
QS(9)=-BYAB
QS(10)=0.
QS(11)=-BYCD
QS(12)=0.
QS(39)=0.
QS(40)=-1.
GO TO 13
11 CONTINUE
QS(17)=BXAB*COS(BXAB*L)
QS(18)=-BXAB*SIN(BXAB*L)
QS(19)=BXCD*COSH(BXCD*L)
QS(20)=BXCD*SINH(BXCD*L)
QS(9)=-BYAB*COS(BYAB*L)
QS(10)=BYAB*SIN(BYAB*L)
QS(11)=-BYCD*COSH(BYCD*L)
QS(12)=-BYCD*SINH(BYCD*L)
QS(39)=-SIN(BP*L)
QS(40)=-COS(BP*L)
13 CALL ADD(-1.,QS,0.,QS,QS)

```

RETURN
END

C

```
SUBROUTINE PQFORM(W,L,E,P,Q,DUM,DUN)
REAL L,LI,MPL,IPL
DIMENSION L(9),E(5),P(5),Q(5),DUM(5),DUN(5)
NBEAM=L(1)+.01
DO 1 IBEAM = 1,NBEAM
LI=L(IBEAM+4)
EIX=E((IBEAM-1)*9+5)
EIY=E((IBEAM-1)*9+6)
EAX=E((IBEAM-1)*9+7)
EIP=E((IBEAM-1)*9+8)
MPL=E((IBEAM-1)*9+9)
IPL=E((IBEAM-1)*9+10)
FZ=E((IBEAM-1)*9+11)
AGKX=E((IBEAM-1)*9+12)
AGKY=E((IBEAM-1)*9+13)
CALL PFORM(W,LI,0.,EIX,EIY,EAX,EIP,MPL,IPL,FZ,AGKX,AGKY,DUM
1,DUN)
IF(IBEAM-1)2,3,2
3 CALL MAKE(P,DUM)
GO TO 4
2 CALL JUXTV(P,DUM,P)
4 CALL JUXTV(P,DUN,P)
CALL QFORM(W,LI,0.,EIX,EIY,EAX,EIP,MPL,IPL,FZ,AGKX,
1AGKY,DUM,DUN)
IF(IBEAM-1)5,6,5
6 CALL MAKE(Q,DUM)
GO TO 7
5 CALL JUXTV(Q,DUM,Q)
7 CALL JUXTV(Q,DUN,Q)
CALL PFORM(W,LI,LI,EIX,EIY,EAX,EIP,MPL,IPL,FZ,AGKX,
1AGKY,DUM,DUN)
CALL JUXTV(P,DUM,P)
CALL JUXTV(P,DUN,P)
CALL QFORM(W,LI,LI,EIX,EIY,EAX,EIP,MPL,IPL,FZ,AGKX,
1AGKY,DUM,DUN)
CALL JUXTV(Q,DUM,Q)
1 CALL JUXTV(Q,DUN,Q)
RETURN
END
```

C

```
707 SUBROUTINE WSEARCH(W,DW,A,DETNEW,DETOLD,WI)
DIMENSION A(5),DETOLD(5),DETNEW(5),WI(5)
FORMAT(7E12.5)
NA=A(1)
A77=A(77)
CALL MAKE(DETOLD,DETNEW)
DETO=1.
DO 3 I =1,NA
3 DETO=DETO*DETOLD(I+4)
CALL UPPER(A,DETNEW)
DETN=1.
DO 1 I=1,NA
1 DETN=DETN*DETNEW(I+4)
FREQ=W/6.2831853
PRINT 707,FREQ,A77,DETN
IF(DETO*DETN)4,2,2
4 WROOT=W-DW-DETO*DW/(DETN-DETO)
NW=WI(1)+1.001
WI(1)=NW
WI(NW+4)=WROOT
CALL SPIT(WI,3H WI)
2 RETURN
END
```

```

C      SUBROUTINE UPPER(A,DETA)
C      GERNERATES THE DETERMINANT, DETA, OF MATRIX A
101    DIMENSION A(5),DETA(5)
      FORMAT(2E15.7)
      N = A(1) + .01
      NM1 = N - 1
      NP1 = N + 1
      DO 5 K = 1,NM1
      KROW=K
      AKK=A(K*N+K-N+4)
      IF(AKK)1,2,1
2      KROW=KROW+1
      IF(KROW-N)7,7,5
7      AKK=A(KROW*N+K-N+4)
      IF(AKK)3,2,3
3      DO 4 J=K,N
      AD=A(K*N+J-N+4)
      A(K*N+J-N+4)=A(KROW*N+J-N+4)
4      A(KROW*N+J-N+4)=AD
1      AKK1 = 1./AKK
      K1 = K + 1
      DO 6 KI = K1,N
      AKIK = A(KI*N + K - N+4)*AKK1
      A(KI*N + K - N + 4)=0.
      DO 6 L = K1,N
      KIL = KI*N + L - N + 4
      A(KIL) = A(KIL) - AKIK*A(K*N + L - N + 4)
6      CONTINUE
5      CONTINUE
      AD=1.
      CALL SET(DETA,N,1)
      DO 16 I = 1,N
      DETA(I+4)=A(I*N + I - N + 4)
      EYE=I
      AD=AD*DETA(I+4)
      IF(AD)26,27,26
27     PRINT 101,AD,EYE
      AD=1.
26     CONTINUE
16     CONTINUE
      RETURN
      END

```

```

C      SUBROUTINE DIAG(A,SHAPE)
123    DIMENSION A(5),SHAPE(5)
      FORMAT(I10,5E12.4)
      NA=A(1)+.01
      NAM1=NA-1
      DO 1 I = 1,NAM1
      IP1=NA-I+1
      AD=0.
      DO 2 J = IP1,NA
2      AD=AD-SHAPE(J+4)*A((NA-I-1)*NA+J+4)
      SHAPE(NA-I+4)=AD/A((NA-I-1)*NA+NA-I+4)
1      CONTINUE
      RETURN
      END

```

Reference Paper 3:

PDEMOD -

**A Computer Program for Distributed Parameter Estimation
of Flexible Aerospace Structures,
Part I: Theory and Verification**

**PDEMOD - A Computer Program
for Distributed Parameter Estimation of Flexible Aerospace Structures
Part I: Theory and Verification**

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ABSTRACT

During the past three decades the finite element method matured gradually and dominated almost exclusively all the engineering applications. But the practice of generating complex finite element dynamic models of aerospace structures has revealed a number of shortcomings. First, the high dimensionality of the models requires an order-reduction process before a control system can be designed. However, seemingly unimportant modes can be inadvertently eliminated which prove later to be significant to control system performance and stability. Second, use of finite element models for dynamic system identification is generally very difficult because of the extremely large amount of unknowns. Third, the structural damping is generally added ad hoc after generating an undamped model. Inaccurate damping results, especially for the modes which couple different types of motion.

In contrast, distributed parameter models offer an alternative to finite element models for overall dynamic analysis and control synthesis for aerospace and aeronautical structures. First, the modal order does not have to be reduced prior to the inclusion of control system dynamics, which eliminates the risk involved with modal truncation. Second, distributed parameter models inherently involve fewer parameters, thereby enabling more accurate parameter estimation using experimental data. Third, it is possible to include the damping in the basic model, thereby increasing the accuracy of the structural damping. Recently, distributed parameter models have

been made for some large space structures. Distributed parameter models may provide an efficient complementary methodology to the finite element approach.

The software package PDEMODO was initialized by the late Lawrence W. Taylor, Jr. at NASA Langley Research Center during the middle of the 1980's. The initial interest in the package PDEMODO was to model the structural dynamics of general spacecraft configurations by using the distributed parameter approach, which consists of a three-dimensional network of flexible beams and rigid bodies. The building blocks from which three-dimensional configurations can be constructed consist of (1) beams, which have bending in two directions, torsion, and elongation degrees of freedom, and (2) rigid bodies, which are connected by any network of beam elements. The full six degrees of freedom are allowed at either end of the beam. Rigid bodies can be attached to the beam at any angle or body location. The modified Bernoulli-Euler beam equation is used to represent the bending, the wave equations for torsion and elongation.

A system of partial differential equations (PDEs) is formulated and connected at the elements' boundaries based on the compatibility conditions. The equations of motion for any number of rigid bodies are written in the frequency domain and in terms of the coefficients of the sinusoidal and hyperbolic functions which comprise the mode shapes. The force and moment vectors for both ends of a single beam element can be described in terms of the spatial derivatives of the solutions of the corresponding PDE's. Distributed parameter models can therefore be generated for any three-dimensional configurations describable by PDEs joined at their boundaries. Because of Mr. Taylor's sudden demise, it becomes an urgent task to summarize and sift his research achievements and make them available to the other researchers. We have continued his work and recovered the basic functions of this package which may provide an opportunity to more researchers to apply distributed parameter modeling techniques to a variety of aerospace structures. The verification of the code has been conducted by comparing the results with those examples for which the exact theoretical solutions can be obtained.

By investigating the potential of the distributed parameter modeling technique, we expect that the PDEMODO may be further developed in the following aspects: (1) structural dynamics, modal frequencies and mode shapes; (2) parameter estimation of modal characteristics; (3) structural damping; (4) control system dynamics; and (5) design optimization. In its present stage, however, only the first of the functions has been completed and included in the package. Meanwhile, a massive effort is being conducted which is expected to modify the mathematical model and global system generating procedure to develop the methodology for control analysis purpose. Instead of using the coefficients of the solution functions, the transfer matrix may be used finally, which provides a much more convenient way to describe the state-vector transition from one point of the structure to the other. All these research outputs are planned to be contents of the modified version of the package.

INTRODUCTION

During the past three decades the finite element method had become extremely popular in a very wide field of engineering. As early as the 1960's, the aircraft industry had developed in-house finite element programs. During the 1970's, some general-purpose finite element programs such as NASTRAN were released for public use, bringing with them a significant technology base that led to development of numerous commercial finite element software systems. Later, the

various commercial packages were refined, and their technology base was expanded. NASA's research in computational structures technology (CST)_[1] is helping to develop the finite element analysis to a new stage that is capable of using a general automated unstructured grid generation to discretize the aerodynamic field and the structure for thermal and structural analysis_[2].

But such a treatment having a very large number of elements is usually very expensive and time consuming. In static finite element analyses, models having over 10,000 degrees of freedom (DOF) are not uncommon. In fact, a finite element model of a helicopter fuselage currently under design contains about 27,000 grid points and 149,000 DOF. It is economically unfeasible and normally unnecessary to conduct dynamic analysis with this many unknowns. Most of the dynamic models based on the finite element approach must resort to modal truncation techniques to reduce the number of unknowns prior to dynamic analysis. However, the control spillover resulting from the modal truncation may lead to degradation of control system performance, or even create instability_[3].

The configuration of large space structures usually has the following common characteristics: extremely large dimension, light-weight design, high flexibility, rather uniform mass and stiffness distribution, low and closely spaced natural frequencies, and slight and/or improperly modeled damping. Structures with these characteristics are essentially distributed parameter systems by nature, and should be most accurately modeled as a continuously distributed mass and stiffness over the entire structural area rather than a sequence of finite mass elements coupled together as the finite element approach does.

In contrast, distributed parameter modeling provides a very practical approach for overall dynamic analysis and control synthesis for aerospace and aeronautical structures. Recently, distributed parameter models have been made for some large space structures, such as, the Spacecraft Control Laboratory Experiment (SCOLE)_[4], Solar Array Flight Experiment (SAFE)_[5], Space Station, Freedom_[6], Low-power Atmospheric Compensation Experiment (LACE) satellite model_[7], and the Aerospace Large Flexible Manipulator_[8]. The fundamental advantage of using the distributed parameter approach is to decrease the number of unknowns significantly. In the preliminary design stage, the detailed structural design is often neglected; therefore, a simple but efficient global structural model may be more beneficial to weigh the trade-off between the performance and cost, between the structural penalty and control system consummation, etc. Distributed parameter models may provide an efficient complementary methodology to the finite element approach.

The software package PDEMOD was initialized by the late Lawrence W. Taylor, Jr. at NASA Langley Research Center during the middle of the 1980's. The first release of his work on PDEMOD package was in 1987_[9,10,11]. The initial interest in the package PDEMOD was to model the structural dynamics of general spacecraft configurations by using the distributed parameter approach, which consists of three-dimensional network of flexible beams and rigid bodies. The building blocks from which three-dimensional configurations can be constructed consist of (1) beams, which have bending in two directions, torsion, and elongation degrees of freedom, and (2) rigid bodies, which are connected by any network of beam elements. The full six degrees of freedom are allowed at either end of the beam. Rigid bodies can be attached to the beam at any angle or body location. The modified Bernoulli-Euler beam equation is used to represent the bending, the wave equations for torsion and elongation.

A system of partial differential equations (PDEs) is formulated and connected at the elements' boundaries based on the compatibility conditions. The equations of motion for any

number of rigid bodies are written in the frequency domain and in terms of the coefficients of the sinusoidal and hyperbolic functions which comprise the mode shapes. The force and moment vectors for both ends of a single beam element can be described in terms of the spatial derivatives of the solutions of the corresponding PDE's. Distributed parameter models can therefore be generated for any three-dimensional configurations describable by PDEs joined at their boundaries. The manual labor of generating such models is therefore avoided.

Because of Mr. Taylor's sudden demise, it becomes an urgent task to summarize and sift his research achievements and make them available to the other researchers. We continued his work and recovered the basic functions of this package which may provide an opportunity to more researchers to apply distributed parameter modeling techniques to a variety of aerospace structures. The verification of the code has been conducted by comparing the results with those examples for which the exact theoretical solutions can be obtained, for instance, a simple beam with various boundary conditions, etc.

By investigating the potential of the distributed parameter modeling technique, we expect that the PDEMOD may be further developed in the following aspects: (1) structural dynamics, modal frequencies and mode shapes; (2) parameter estimation of modal characteristics; (3) structural damping; (4) control system dynamics; and (5) design optimization. In its present stage, however, only the first of the functions has been completed and included in the package. Meanwhile, a massive effort is being conducted which is expected to modify the mathematical model and global system generating procedure^[7,12] to develop the methodology for control analysis purpose^[13,14]. Instead of using the coefficients of the solution functions, the transfer matrix may be used finally, which provides a much more convenient way to describe the state-vector transition from one point of the structure to the other. All these research outputs are planned to be contents of the modified version of the package.

PARTIAL DIFFERENTIAL EQUATIONS (PDEs)

A complex large space structure can be decomposed into simple pieces. A network of distributed parameter elements and the attached rigid bodies are connected to represent the structural dynamics of the complex flexible spacecraft. Each flexible beam element exhibits lateral bending u and v in two axes, axial deformation w , and torsion ψ , as shown in Fig.1, which can be independently described by a variety of PDEs^[15].

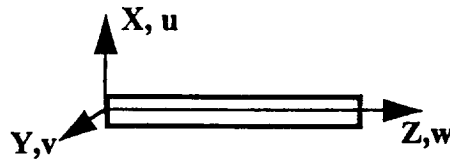


Figure 1 A Beam Element

The bending behavior of a beam element can be described by a modified Bernoulli-Euler beam equation which includes Euler bending stiffness, Timoshenko shear, and axial-force stiffness for lateral deflections in the x-z and y-z planes. The PDE for bending in the x-z plane is

$$m\ddot{u} + EI_x u'''' + GA\ddot{u}' + F_0 u' = q_x(z, t) \quad (1)$$

The corresponding equation for bending in the y-z plane is

$$m\ddot{v} + EI_y v'''' + GA\ddot{v}' + F_0 v' = q_y(z, t) \quad (2)$$

The axial and torsional dynamics are represented by wave equations,

$$m\ddot{w} - EA w'' = F_z(z, t) \quad (3)$$

and

$$J_\psi \ddot{\psi} - GI_\psi \psi'' = M_z(z, t) \quad (4)$$

respectively. The PDEs provide the relationships between the modal frequency and the eigenvalues for the mode shape functions. The Euler and wave equations can be solved for the zero damping cases to produce the following relationships between the modal frequency ω and the eigenvalue $\beta_{[16]}$. For bending in the x-z plane,

$$\beta_{x,2}^2 = \pm \frac{1}{2} \left(\frac{m\omega^2}{GA} + \frac{F_0}{EI_x} \right) + \sqrt{\frac{1}{2} \left(\frac{m\omega^2}{GA} + \frac{F_0}{EI_x} \right)^2 + \frac{m\omega^2}{EI_x}} \quad (5)$$

For bending in the y-z plane,

$$\beta_{y,2}^2 = \pm \frac{1}{2} \left(\frac{m\omega^2}{GA} + \frac{F_0}{EI_y} \right) + \sqrt{\frac{1}{2} \left(\frac{m\omega^2}{GA} + \frac{F_0}{EI_y} \right)^2 + \frac{m\omega^2}{EI_y}} \quad (6)$$

For elongation and torsion, we have

$$\beta_z^2 = \frac{m\omega^2}{EA} \quad (7)$$

and

$$\beta_\psi^2 = \frac{J_\psi \omega^2}{GI_\psi} \quad (8)$$

respectively.

MODE SHAPE FUNCTIONS

The solutions of these partial differential equations for zero damping produce the sinusoidal and hyperbolic spatial equations which comprise the mode shape functions. For the case that $F_0=0$, the bending mode shape in the x-z plane is,

$$u = A_x \sin \beta_x z + B_x \cos \beta_x z + C_x \sinh \beta_x z + D_x \cosh \beta_x z \quad (9)$$

Similarly, for the bending in the x-z plane the mode shape function is,

$$v = A_y \sin \beta_y z + B_y \cos \beta_y z + C_y \sinh \beta_y z + D_y \cosh \beta_y z \quad (10)$$

The undamped mode shape function for elongation along the z-axis is

$$w = A_z \sin \beta_z z + B_z \cos \beta_z z \quad (11)$$

The undamped mode shape function for torsion about the z-axis is

$$\psi = A_\psi \sin \beta_\psi z + B_\psi \cos \beta_\psi z \quad (12)$$

These undamped mode shape functions are expected to be good approximations to the exact solutions for low level of damping. The mode shape of the entire configuration consists of all these functions, repeated for each beam element. Because of bending in two directions, elongation, and torsion, a total of 12 coefficients are needed for each beam element. A vector of the coefficients of these sinusoidal and hyperbolic functions is defined as the mode shape parameter vector,

$$\{\theta\} = [A_x B_x C_x D_x A_y B_y C_y D_y A_z B_z A_\psi B_\psi]^T \quad (13)$$

The translational deflection vector is defined as,

$$\{U\} = \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = [Q_u(z)] \{\theta\} \quad (14)$$

where,

$$[Q_u(z)] = \begin{bmatrix} Q_u^{11} & Q_u^{12} & Q_u^{13} & Q_u^{14} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_u^{25} & Q_u^{26} & Q_u^{27} & Q_u^{28} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q_u^{39} & Q_u^{3,10} & 0 & 0 \end{bmatrix}$$

the non-zero elements of the matrix $[Q_u(z)]$ are as follows,

$$\begin{array}{llll} Q_u^{11} = \sin \beta_x z & Q_u^{12} = \cos \beta_x z & Q_u^{13} = \sinh \beta_x z & Q_u^{14} = \cosh \beta_x z \\ Q_u^{25} = \sin \beta_y z & Q_u^{26} = \cos \beta_y z & Q_u^{27} = \sinh \beta_y z & Q_u^{28} = \cosh \beta_y z \\ Q_u^{39} = \sin \beta_z z & Q_u^{3,10} = \cos \beta_z z & & \end{array}$$

The angular deflection vector is defined as,

$$\{U'\} = \begin{Bmatrix} u' \\ v' \\ \psi \end{Bmatrix} = [Q_s(z)]\{\theta\} \quad (15)$$

where,

$$[Q_s(z)] = \begin{bmatrix} Q_s^{11} & Q_s^{12} & Q_s^{13} & Q_s^{14} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_s^{25} & Q_s^{26} & Q_s^{27} & Q_s^{28} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q_s^{3,11} & Q_s^{3,12} \end{bmatrix}$$

the non-zero elements of the matrix $[Q_s(z)]$ are as follows,

$$\begin{aligned} Q_s^{11} &= \beta_x \cos \beta_x z & Q_s^{12} &= -\beta_x \sin \beta_x z & Q_s^{13} &= \beta_x \cosh \beta_x z & Q_s^{14} &= \beta_x \sinh \beta_x z \\ Q_s^{25} &= \beta_y \cos \beta_y z & Q_s^{26} &= -\beta_y \sin \beta_y z & Q_s^{27} &= \beta_y \cosh \beta_y z & Q_s^{28} &= \beta_y \sinh \beta_y z \\ Q_s^{3,11} &= \sin \beta_y z & Q_s^{3,12} &= \cos \beta_y z \end{aligned}$$

FORCES AND MOMENTS

Next, it is necessary to express the forces and moments at either end of the beam element in terms of the mode shape parameter vector $\{\theta\}$. The force vector is as

$$\{F\} = \begin{Bmatrix} F_x \\ F_y \\ F_z \end{Bmatrix} = \begin{Bmatrix} EI_x u'' \\ EI_y v'' \\ EA w' \end{Bmatrix} = [P_F(z)]\{\theta\} \quad (16)$$

where,

$$[P_F(z)] = \begin{bmatrix} P_F^{11} & P_F^{12} & P_F^{13} & P_F^{14} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & P_F^{25} & P_F^{26} & P_F^{27} & P_F^{28} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & P_F^{39} & P_F^{3,10} & 0 & 0 \end{bmatrix}$$

the non-zero elements of the matrix $[P_F(z)]$ are as follows,

$$\begin{aligned} P_F^{11} &= -\beta_x^3 EI_x \cos \beta_x z & P_F^{12} &= \beta_x^3 EI_x \sin \beta_x z \\ P_F^{13} &= \beta_x^3 EI_x \cosh \beta_x z & P_F^{14} &= \beta_x^3 EI_x \sinh \beta_x z \\ P_F^{25} &= -\beta_y^3 EI_y \cos \beta_y z & P_F^{26} &= \beta_y^3 EI_y \sin \beta_y z \\ P_F^{27} &= \beta_y^3 EI_y \cosh \beta_y z & P_F^{28} &= \beta_y^3 EI_y \sinh \beta_y z \\ P_F^{39} &= \beta_z EA \cos \beta_z z & P_F^{3,10} &= -\beta_z EA \sin \beta_z z \end{aligned}$$

The moment vector can be expressed as

$$\{M\} = \begin{Bmatrix} M_x \\ M_y \\ M_z \end{Bmatrix} = \begin{Bmatrix} EI_x u'' \\ EI_y v'' \\ GI_\psi \psi' \end{Bmatrix} = [P_M(z)]\{\theta\} \quad (17)$$

where,

$$[P_M(z)] = \begin{bmatrix} P_M^{11} & P_M^{12} & P_M^{13} & P_M^{14} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & P_M^{25} & P_M^{26} & P_M^{27} & P_M^{28} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & P_M^{3,11} & P_M^{3,12} \end{bmatrix}$$

the non-zero elements of the matrix $[P_M(z)]$ are as follows,

$$\begin{aligned} P_M^{11} &= -\beta_x^2 EI_x \sin \beta_x z & P_M^{12} &= -\beta_x^2 EI_x \cos \beta_x z \\ P_M^{13} &= \beta_x^2 EI_x \sinh \beta_x z & P_M^{14} &= \beta_x^2 EI_x \cosh \beta_x z \\ P_M^{25} &= -\beta_y^2 EI_y \sin \beta_y z & P_M^{26} &= -\beta_y^2 EI_y \cos \beta_y z \\ P_M^{27} &= \beta_y^2 EI_y \sinh \beta_y z & P_M^{28} &= \beta_y^2 EI_y \cosh \beta_y z \\ P_M^{3,11} &= \beta_\psi GI_\psi \cos \beta_\psi z & P_M^{3,12} &= -\beta_\psi GI_\psi \sin \beta_\psi z \end{aligned}$$

It is also necessary to account for changes in axes from each beam to the body to which it is attached, and for points of attachment at some distance from the center of gravity of the body. The forces and moments that the beam i applies to the body j expressed in the body j 's coordinate system are,

$$\{F\}_{ji} = [T]_{ji} \{F\}_{ij} = [T]_{ji} [P_F]_i \{\theta\}_i \quad (18)$$

$$\{M\}_{ji} = [T]_{ji} (\{M\}_{ij} + [R]_{ji} \{F\}_{ij}) = [T]_{ji} ([P_M]_i + [R]_{ji} [P_F]_i) \{\theta\}_i \quad (19)$$

where, the coordinate-transformation matrix from the beam i 's coordinate system to the body j 's coordinate system is

$$[T]_{ji} = \begin{bmatrix} \cos(X_j, x_i) & \cos(X_j, y_i) & \cos(X_j, z_i) \\ \cos(Y_j, x_i) & \cos(Y_j, y_i) & \cos(Y_j, z_i) \\ \cos(Z_j, x_i) & \cos(Z_j, y_i) & \cos(Z_j, z_i) \end{bmatrix} \quad (20)$$

the eccentric matrix at the attachment point between the beam i and body j is

$$[R]_{ji} = \begin{bmatrix} 0 & -r_z & r_y \\ r_z & 0 & -r_x \\ -r_y & r_x & 0 \end{bmatrix}_{ji} \quad (21)$$

RIGID BODY MOTIONS

A Newtonian or inertial frame of reference $X_0Y_0Z_0$ is used for the motions of all beam elements and rigid bodies.

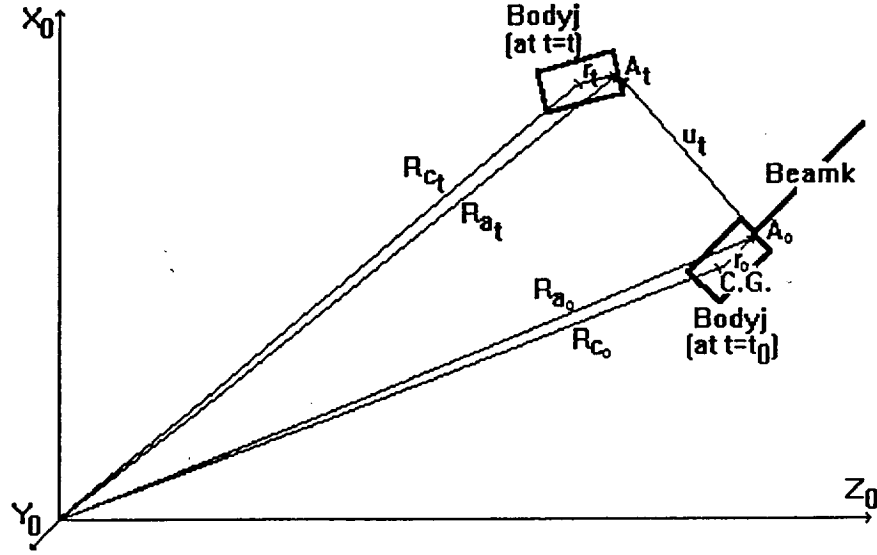


Figure 2 Rigid Body Motion

Consider body j connecting several beams, of which beam k is taken into account as a datum. From Fig.2, we have

$$\text{at time } t=t_0: \quad \bar{R}_{a_0} = \bar{R}_{c_0} + \bar{r}_0$$

$$\text{at time } t=t: \quad \bar{R}_{a_t} = \bar{R}_{a_0} + \bar{u}_t = \bar{R}_{c_0} + \bar{r}_0 + \bar{u}_t$$

On the other hand, $\bar{R}_{a_t} = \bar{R}_{c_t} + \bar{r}_t$, so,

$$\bar{R}_{c_t} = \bar{R}_{a_t} - \bar{r}_t = \bar{R}_{c_0} + \bar{r}_0 + \bar{u}_t - \bar{r}_t = \bar{R}_{c_0} + \bar{u}_t + \bar{r}_0 \times \bar{u}'_t \quad (22)$$

where, \bar{u}_t and \bar{u}'_t are the translational and angular deflection vectors at the attachment point between the body j and the beam k , respectively, and the vector difference $\bar{r}_0 - \bar{r}_t$ has been expressed as the vector cross product of \bar{r}_0 and \bar{u}'_t , i.e. $\Delta \bar{r} = \bar{r}_0 - \bar{r}_t = \bar{r}_0 \times \bar{u}'_t$, which can also be written in matrix form as

$$\begin{Bmatrix} \Delta r_x \\ \Delta r_y \\ \Delta r_z \end{Bmatrix} = \begin{bmatrix} 0 & -r_z & r_y \\ r_z & 0 & -r_x \\ -r_y & r_x & 0 \end{bmatrix} \begin{Bmatrix} u'_x \\ u'_y \\ \psi \end{Bmatrix} \quad (23)$$

Therefore, Eq.22 can be written in matrix form as

$$\{R_{CG}\}_{j_i} = \{R_{CG}\}_{j_0} + [T]_{jk} \{u\}_{jk_i} + [R]_{jk} [T]_{jk} \{u'\}_{jk_i} \quad (24)$$

Differentiating Eq.24, we get the acceleration of the body j's center of gravity (C.G.),

$$\{\ddot{R}_{CG}\}_j = [T]_{jk} \{\ddot{u}\}_{jk} + [R]_{jk} [T]_{jk} \{\ddot{u}'\}_{jk} \quad (25)$$

The angular acceleration of the body j is simply expressed as

$$\{\varepsilon\}_j = [T]_{jk} \{\ddot{u}'\}_{jk} \quad (26)$$

STRUCTURAL DYNAMIC EQUATIONS

The equations of motion for the connected bodies and elements consist of blocks of terms, assembled in an order dictated by the body and beam indices. The mass times the acceleration of each body is related to the sum of forces caused by each beam element and each applied force.

$$[m]_j \{\ddot{R}_{CG}\}_j = \sum_i \{F\}_{ji} + \sum_m \{f\}_{jm} + \{g\} \quad (27)$$

where, $\sum_i \{F\}_{ji}$ is the sum of the i-beam forces acting on the body j; $\sum_m \{f\}_{jm}$ is the sum of the m-applied forces acting on the body j; $\{g\}$ is the gravitational vector. It is similar for the moment equation,

$$[J]_j \{\varepsilon\}_j = \sum_i [\{M\}_{ji} + [T]_{ij} [R]_{ji} [T]_{ji} \{F\}_{ji}] + \sum_n \{M\}_{jn} + \sum_m [T]_{mj} [R]_{jm} [T]_{jm} \{f\}_{jm}] \quad (28)$$

where, $\sum_i \{M\}_{ji}$ is the sum of the i-beam moments acting on the body j; $\sum_i [T]_{ij} [R]_{ji} [T]_{ji} \{F\}_{ji}$ is the sum of the moments acting on the body j caused by the i-beam forces $\{F\}_{ji}$; $\sum_n \{M\}_{jn}$ is the sum of the n-applied moments acting on the body j; and $\sum_m [T]_{mj} [R]_{jm} [T]_{jm} \{f\}_{jm}$ is the sum of the moments caused by the m-applied forces acting on the body j.

For the case of without applied forces and moments and neglecting gravity force, referring to Eqs.25 and 26, we derive the following two equations. From the force equation,

$$[m]_j ([T]_{jk} \{\ddot{u}\}_{jk} + [R]_{jk} [T]_{jk} \{\ddot{u}'\}_{jk}) = \sum_i \{F\}_{ji} \quad (29)$$

From the moment equation,

$$[J]_j [T]_{jk} \{\ddot{u}'\}_{jk} = \sum_i [\{M\}_{ji} + [T]_{ij} [R]_{ji} [T]_{ji} \{F\}_{ji}] \quad (30)$$

Let us express both sides of Eqs.29 and 30 in terms of the mode shape parameter vector, then,

$$-\omega^2[m]_j([T]_{jk}[Q_u]_k + [R]_{jk}[T]_{jk}[Q_s]_k)\{\theta\}_k = \sum_i [T]_{ji}[P_F]_i\{\theta\}_i \quad (31)$$

and

$$-\omega^2[J]_j[T]_{jk}[Q_s]_k\{\theta\}_k = \sum_i ([T]_{ji}[P_M]_i + [R]_{ji}[T]_{ji}[P_F]_i)\{\theta\}_i \quad (32)$$

Eqs.31 and 32 are the structural dynamic equations for the most general configurations. To demonstrate the overall procedure more clearly, let's assume that there is only one beam element attached to the body j, that is, $i=k=1$. In this case, Eqs.31 and 32 will be simplified as

$$\begin{matrix} [A_F] \\ 3 \times 12 \end{matrix} \begin{matrix} \{\theta\}_i \\ 12 \times 1 \end{matrix} = \{0\} \quad (33-1)$$

$$\begin{matrix} [A_M] \\ 3 \times 12 \end{matrix} \begin{matrix} \{\theta\}_i \\ 12 \times 1 \end{matrix} = \{0\} \quad (33-2)$$

where,

$$\begin{aligned} [A_F] &= [T]_{ji}[Q_u]_i + [R]_{ji}[T]_{ji}[Q_s]_i + \frac{1}{\omega^2}[m]_j^{-1}[T]_{ji}[P_F]_i \\ [A_M] &= [T]_{ji}[Q_s]_i + \frac{1}{\omega^2}[J]_j^{-1}([T]_{ji}[P_M]_i + [R]_{ji}[T]_{ji}[P_F]_i) \end{aligned}$$

Two equations in Eq.33 may be combined as

$$\begin{matrix} [A] \\ 6 \times 12 \end{matrix} \begin{matrix} \{\theta\}_i \\ 12 \times 1 \end{matrix} = \{0\} \quad (34)$$

where the system matrix $[A]$ consists of the two block matrices $[A_F]$ and $[A_M]$. We can see that the number of equations is less than the number of unknown parameters. The difference is six. This is because there are six rigid-body degrees of freedom (d.o.f.s) for this particular one-body-one-beam system. We should have, therefore, six constraint equations to fix the six rigid-body d.o.f.s. For most common cases, six boundary conditions provide six constraint equations, which can also be expressed in terms of the mode shape parameter vector $\{\theta\}_i$, but the format of the equations depends on what the specific boundary conditions are. Superimposing the constraint equations into Eq.34, we obtain a new system dynamic equation

$$\begin{matrix} [\bar{A}] \\ 12 \times 12 \end{matrix} \begin{matrix} \{\theta\}_i \\ 12 \times 1 \end{matrix} = \{0\} \quad (35)$$

which has a full-rank system matrix $[\bar{A}]$ for any values of circular frequencies except the natural frequencies. Based on the condition that $Det[\bar{A}] = 0$, the natural frequencies of the system can be determined.

For general configurations, the structural dynamic equation may become very complicated, but the procedure to generate the equation is the same as stated. In general, for a structural system consisting of J bodies and I beams, the structural dynamic equation has the form of

$$[A]_{(6 \times J) \times (12 \times J)} \{\theta\}_i = \{0\}_{(12 \times J) \times 1} \quad (36)$$

The difference between the number of unknowns and the number of equations is exactly equal to the number of rigid-body d.o.f.s. To fix the rigid-body d.o.f.s, it is sometimes necessary to consider the compatibility conditions besides the boundary conditions. The following two special cases must be paid more attention.

(1) For the case that a rigid body may have more than one beam element attached as shown in Fig.3 where two beams are attached to the body j, additional constraint equations must be added to the system equation, Eq.36, which appears as for this particular example in the figure,

$$[A]_{12 \times 24} \{\theta\}_{24 \times 1} = \{0\}_{24 \times 1} \quad (37)$$

It is clear that 12 extra equations are needed to fit the two bodies' rigid-body d.o.f.s. While the boundary conditions provides six equations, the other six equations must be found from the compatibility conditions.

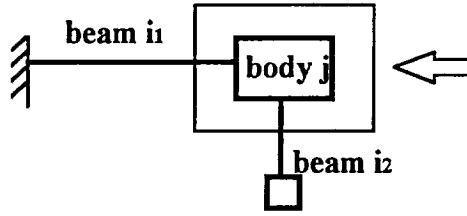


Figure 3 A Rigid Body Attached by Two Beam Elements

To account for the continuity in translational deflection at the two attachment points, the constraint equation must be satisfied,

$$[T]_{j,i1} \{u\}_{j,i1} - [R]_{j,i1} [T]_{j,i1} \{u'\}_{j,i1} = [T]_{j,i2} \{u\}_{j,i2} - [R]_{j,i2} [T]_{j,i2} \{u'\}_{j,i2} \quad (38)$$

The constraint equation for ensuring continuity in the angular deflection at the two attachment points is

$$[T]_{j,i1} \{u'\}_{j,i1} = [T]_{j,i2} \{u'\}_{j,i2} \quad (39)$$

Expressing Eqs.38 and 39 in terms of the mode shape parameter vectors of the two beams, we have

$$([T]_{j,i1} [Q_u]_{i1} - [R]_{j,i1} [T]_{j,i1} [Q_s]_{i1}) \{\theta\}_{i1} = ([T]_{j,i2} [Q_u]_{i2} - [R]_{j,i2} [T]_{j,i2} [Q_s]_{i2}) \{\theta\}_{i2} \quad (40)$$

and

$$[T]_{j,i1} [Q_s]_{i1} \{\theta\}_{i1} = [T]_{j,i2} [Q_s]_{i2} \{\theta\}_{i2} \quad (41)$$

Combining Eqs.40 and 41, we find the six additional equations.

(2) For the case that a beam connects two rigid bodies at its two ends as shown in Fig.4 where beam i connects two bodies. More discussion must be addressed for this case. Apparently, it seems that the system equation, Eq.36, had a full-rank system matrix, since Eq.36 appears as for this particular example in the figure,

$$\begin{bmatrix} A \end{bmatrix}_{12 \times 12} \{ \theta \}_i = \{ 0 \}_{12 \times 1} \quad (42)$$

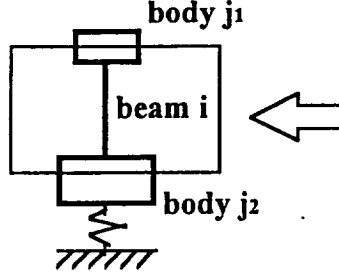


Figure 4 A Beam Element Connecting Two Rigid Bodies

But, it is wrong. The problem is that body j_1 and body j_2 connect to the beam i at different attachment points. Looking back to Eqs.14 to 17, and 31, we find that the components of the system matrix $[A]$, such as $[Q_u(z)]$, $[Q_s(z)]$, $[P_F(z)]$, $[P_M(z)]$, are functions of the beam's longitudinal coordinate z . Based upon the connection between body j_1 and beam i, a set of equations, which is the same as Eq.34, can be found,

$$\begin{bmatrix} A_{j1}(z_i = 0) \end{bmatrix}_{6 \times 12} \{ \theta \}_i = \{ 0 \}_{12 \times 1} \quad (43-1)$$

Similar equation exists between body j_2 and beam i,

$$\begin{bmatrix} A_{j2}(z_i = L_i) \end{bmatrix}_{6 \times 12} \{ \theta \}_i = \{ 0 \}_{12 \times 1} \quad (43-2)$$

The two equations in Eq.43 are not, however, independent since the system matrices $[A_{j1}]$ and $[A_{j2}]$ are both related to the same beam element, beam i. It can be proven that these two matrices are related by a constant matrix $[\Phi]$ which can be derived from beam i's PDEs, that is,

$$\begin{bmatrix} A_{j2}(z_i = L_i) \end{bmatrix} = [\Phi] \begin{bmatrix} A_{j1}(z_i = 0) \end{bmatrix} \quad (44)$$

We can only, therefore, choose one set of equations from Eq.43, and the other six equations must be fixed by the corresponding boundary conditions.

VERIFICATION EXAMPLES

1. EXAMPLE 1: Bending of a Cantilevered Beam.

Length of the Beam $L=130.0$ in.
Bending Stiffness $EI=4 \times 10^7$ Lb.in².
Mass per Length $m=0.09556$ Lb.sec²/in.

Theoretical formula for circular natural frequency: $\omega_n = a_n \sqrt{\frac{EI}{mL^4}} = 1.2106a_n$. The coefficients a_n , and the theoretical natural frequencies and the corresponding results from PDEMOD are listed in Table 1.

Table 1 Results of Example 1

No. of Modes	a_n	Natural Frequency		
		Theoretical Value		PDEMOD
		ω_n	f_n , Hz.	f_n , Hz.
1	3.52	4.2613	0.6782	0.6775
2	22.0	26.6332	4.2388	4.246
3	61.7	74.6940	11.8879	11.890
4	121.0	146.4826	23.3134	22.570
5	200.0	242.1200	38.5346	38.490

2. EXAMPLE 2: Bending of a Clamped-Clamped Beam.

Length of the Beam $L=130.0$ in.
Bending Stiffness $EI=4 \times 10^7$ Lb.in².
Mass per Length $m=0.09556$ Lb.sec²/in.

Theoretical formula for circular natural frequency: $\omega_n = a_n \sqrt{\frac{EI}{mL^4}} = 1.2106a_n$. The coefficients a_n , and the theoretical natural frequencies and the corresponding results from PDEMOD are listed in Table 2.

Table 2 Results of Example 2

No. of Modes	a_n	Natural Frequency		
		Theoretical Value		PDEMOD
		ω_n	f_n , Hz.	f_n , Hz.
1	22.0	26.6332	4.2388	4.31
2	61.7	74.6940	11.8879	11.88
3	121.0	146.4826	23.3134	23.29
4	200.0	242.1200	38.5346	38.50
5	298.2	361.0009	57.4551	57.50

3. EXAMPLE 3: Bending of a Cantilevered Beam with a Tip-Mass M.

Length of the Beam $L=3.077$ ft.
Bending Stiffness $EI=175.9644$ Lb.ft².

Mass per Length $m=0.037 \text{ Lb.sec}^2/\text{ft (slug)}$.
Mass at the Tip $M=0.1538 \text{ Lb.sec}^2/\text{ft (slug)}$.
Equivalent Stiffness of the Beam $k=3EI/L^3=18.1202 \text{ Lb/ft}$.

Theoretical formula for the first circular natural frequency: $\omega_1 = \sqrt{\frac{k}{M+0.23m}}$. The theoretical natural frequency and the corresponding result from PDEMODO are listed in Table 3.

Table 3 Results of Example 3

No. of Modes	Natural Frequency		
	Theoretical Value		PDEMODO
	ω_n	$f_n, \text{Hz.}$	$f_n, \text{Hz.}$
1	10.566	1.682	1.736

4. EXAMPLE 4: Torsion of a Cantilevered Beam.

Length of the Beam $L=130.0 \text{ in.}$
Torsional Stiffness $GI_\psi=4 \times 10^7 \text{ Lb.in}^2$.
Polar Moment of Inertia per Length $J_\psi/L=0.2907 \text{ Lb.sec}^2$.

Theoretical formula for the circular natural frequency: $\omega_n = n\pi \sqrt{\frac{GI_\psi}{(J_\psi/L)L^2}} = 283.4745n$. The theoretical natural frequency and the corresponding result from PDEMODO are listed in Table 4.

Table 4 Results of Example 4

No. of Modes	Natural Frequency		
	Theoretical Value		PDEMODO
	ω_n	$f_n, \text{Hz.}$	$f_n, \text{Hz.}$
1	283.4745	45.1164	45.10
2	566.9490	90.2328	90.19

CONCLUDING REMARKS

The computer software package PDEMODO is being developed to model the structural dynamics of general spacecraft configurations by using the distributed parameter approach. This paper provides the detailed description of the theoretical background used in the PDEMODO formulation. A complex large space structure is considered as an assembly of flexible beam elements and rigid bodies. Each flexible beam element is represented by four independent partial differential equations which exhibit lateral bending in two axes, axial deformation, and torsion. A system of partial differential equations is then formulated and connected at the elements' boundaries based on the compatibility conditions. The equations of motion for any number of rigid bodies are written in the frequency domain and in terms of the mode shape parameter coefficients. The deflections, forces and moments for both ends of a single beam element can be described in terms of the spatial derivatives of the solutions of the corresponding PDE's, further

expressed in terms of the same set of mode shape parameter coefficients. Distributed parameter models can therefore be generated for any three-dimensional configurations describable by PDEs joined at their boundaries. The verification of the code has been conducted by comparing the results with those examples for which the exact theoretical solutions can be obtained.

By investigating the potential of the distributed parameter modeling technique, we expect that the PDEMOD may be further developed in the following areas: (1) structural dynamics, modal frequencies and mode shapes; (2) parameter estimation of modal characteristics; (3) structural damping; (4) control system dynamics; and (5) design optimization. In present stage, however, only the first of these functions has been completed and included in the package. Meanwhile, a massive effort is being conducted which is expected to modify the mathematical model and global system generating procedure to develop methodology for the control analysis purpose. Instead of using the coefficients of the solution functions, the transfer matrix may finally be used, which provides a much more convenient way to describe the state-vector transition from one point of the structure to the other. All these research outputs are planned to be contents of the modified version of the package. It is also necessary to conduct additional testing to establish the accuracy of modeling different types of real space structures so as to move the approach from academic curiosity to a practical alternative for engineering design.

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Reference Paper 4:

**A Method of Superposing
Rigid-Body Kinematics and Flexible Deflection
for End-Effector Vibration Suppression
of a Large Flexible Manipulator System**

A Method of Superposing Rigid-Body Kinematics and Flexible Deflection for End-Effector Vibration Suppression of a Large Flexible Manipulator System

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ABSTRACT

The modeling, analysis and control of a small manipulator system are simplified by considering the links as rigid bodies. For large flexible manipulator systems, however, the flexibility of the links and the joint compliance must be considered. To describe the kinematics and the dynamic behavior of a flexible manipulator system, the common approach is to use Lagrange's equations for both the rigid-body degrees of freedom (d.o.f.'s) and the dynamic deflection d.o.f.'s caused by the flexibility. The generalized coordinates are associated with the rigid-body d.o.f.'s of the links, and the modal coordinates associated with the flexibility d.o.f.'s. The consequence is that a set of highly-coupled and non-linear simultaneous partial differential equations is generated. These equations are so complex and lengthy that it is extremely difficult, if not impossible, to expand them manually even for a lower degree-of-freedom manipulator with a lower number of modes assumed. For the flexible manipulators with greater complexity, the dynamic analysis is literally forbidden by any practical manual symbolic derivations. The computer symbolic derivation of flexible manipulator dynamics was then suggested by several researchers.

For simplifying the analytical process to a realistically acceptable extent, this paper conceives a new mathematical treatment for dynamic analysis of large flexible manipulator systems. The essence of the idea is to separate the kinematics and flexibility analyses as two independent but successive steps in a small time interval. Superposing the kinematic result and the flexibility effect, the summation is viewed as the initial conditions of the next instant motion, and the dynamic analysis succeeds to the next time interval. Repeating this process, the kinematic analysis accompanying flexibility effect is accomplished for a required time period. As usual, the kinematic analysis is based upon the rigid-body link assumption, and the Lagrange's equations are set up for these less amount of rigid-body d.o.f.'s, which are manually manipulative. The flexibility analysis for certain configuration of the manipulator system is conducted by using the distributed parameter system approach, along with the application of the transfer matrix method. Since an extremely complex analytical chore is resolved into two relatively simpler problems, the complexity of the dynamic analysis of large flexible manipulator systems is mathematically simplified. To demonstrate the applicability of the proposed methodology, an end-effector vibration suppression problem for a large manipulator system has been investigated. The manipulator system studied in this paper is a similitude of a NASA manipulator testbed for the research of the berthing operation of the Space Shuttle to the Space Station. The computational

results show that the proposed method is very effective for end-effector vibration suppression of a large flexible manipulator system.

1. INTRODUCTION

The modeling, analysis and control of a small manipulator system are simplified by considering the links as rigid bodies_[1-4]. For large flexible manipulator systems, however, the flexibility of the links and the joint compliance must be considered. The limitations of the rigid link assumption in the formulation and analysis of large flexible manipulator dynamics were investigated extensively. Several formulations can be found in the robotics literature, such as, recursive or non-recursive Lagrangian assumed mode_[5-7], generalized Newton-Euler method_[8], and Lagrangian using Raleigh-Ritz method_[9], etc. To describe the kinematics and the dynamic behavior of a flexible manipulator system, the common approach is to use Lagrange's equations for both the rigid-body degrees of freedom (d.o.f.'s) and the dynamic deflection d.o.f.'s caused by the flexibility. The generalized coordinates are associated with the rigid-body d.o.f.'s of the links, and the modal coordinates associated with the flexibility d.o.f.'s. The consequence is that a set of highly-coupled and non-linear simultaneous partial differential equations is generated. These equations are so complex and lengthy that it is extremely difficult, if not impossible, to expand them manually even for a lower degree-of-freedom manipulator with a lower number of modes assumed. For the flexible manipulators with greater complexity, the dynamic analysis is literally forbidden by any practical manual symbolic derivations. The computer symbolic derivation of flexible manipulator dynamics was later suggested by several researchers. Some of them wrote a symbolic manipulation program_[6,7], some of them suggested using the MATHEMATICA commercial software package_[10]. The basic functions of the symbolic manipulation program may include symbolic simplification of polynomials and rational expressions, linearization of trigonometric functions, automated evaluation of the relative significance of terms and neglecting the less significant terms, and even symbolic integration and differentiation. The application of computer symbolic derivation techniques alleviates the difficulty in the dynamic analysis of large flexible manipulator systems.

For simplifying the analytical process to a realistically acceptable extent, this paper conceives a new mathematical treatment for dynamic analysis of large flexible manipulator systems. The essence of the idea is to separate the kinematics and flexibility analyses as two independent but successive steps in a small time interval. Superposing the kinematic result and the flexibility effect, the summation is viewed as the initial conditions of the next instant motion, and the dynamic analysis succeeds to the next time interval. Repeating this process, the kinematic analysis accompanying flexibility effect is accomplished for a required time period. As usual, the kinematic analysis is based upon the rigid-body link assumption, and the Lagrange's equations are set up for these less amount of rigid-body d.o.f.'s, which are manually manipulative. The flexibility analysis for certain configuration of the manipulator system is conducted by using the distributed parameter system approach, along with the application of the transfer matrix method_[11]. Since an extremely complex analytical chore is resolved into two relatively simpler problems, the complexity of the dynamic analysis of large flexible manipulator systems is mathematically simplified.

To demonstrate the applicability of the proposed methodology, an end-effector vibration suppression problem for a large manipulator system has been investigated. The manipulator

system studied in this paper is a similitude of a NASA manipulator testbed for the research of the berthing operation of the Space Shuttle to the Space Station_[12]. The system studied consists of two flexible links and three revolute joints which is assumed to be constrained in the vertical plane. There are only two rigid-body d.o.f.'s for this specific system. The two Lagrange's equations are set up for the kinematic analysis, and the Runge-Kutta method is used to solve these non-linear partial differential equations numerically. The flexibility of the two links includes the lateral bending and axial elongation. The joint compliance is characterized by its torsional stiffness coefficient. The transfer matrices for the flexible arms and revolute joints have been constructed based on the partial differential equations. According to the compatibility conditions at the connecting points, the global system dynamic equation can be derived. From the corresponding boundary conditions, the characteristic equation for the global system is determined, from which the natural frequencies and mode shape functions can be found. Therefore, the transient response can be obtained. Joint moments are used as both displacement and control actuators. Control law computation proceeds in the frequency domain based on the pole-placement method_[13]. The computational results show that the proposed method is very effective for end-effector vibration suppression of a large flexible manipulator system.

2. TWO-ARM FLEXIBLE MANIPULATOR SYSTEM

The manipulator system (Figure 1) studied in this paper is a similitude of a NASA manipulator testbed for the research of the berthing operation of the space shuttle to the space station (Figure 2). This research testbed is planned to be the model of the berthing process constrained to move in the horizontal plane. Figure 2 illustrates the principal components of the facility. The Space Station Freedom (SSF) Mobility Base is an existing Marshall Space Flight Center (MSFC) Vehicle that has a mass of 2156.4kg. It represents a Space Station in the berthing operation. This vehicle is suspended on the MSFC flat floor facility using low flow-rate air bearings. The flexible appendage shown on the sketch will simulate solar panel disturbances. The vehicle has cold gas reaction jets to allow translational maneuvering. It also has a single gimbal for attitude control. The other vehicle, the Space Shuttle (SS) Mobility Base, is to be of similar construction and will be attached to the SSF Mobility Base with a flexible, two-link manipulator arm. The joints of the arms are driven by electric motors and are suspended by air bearings.

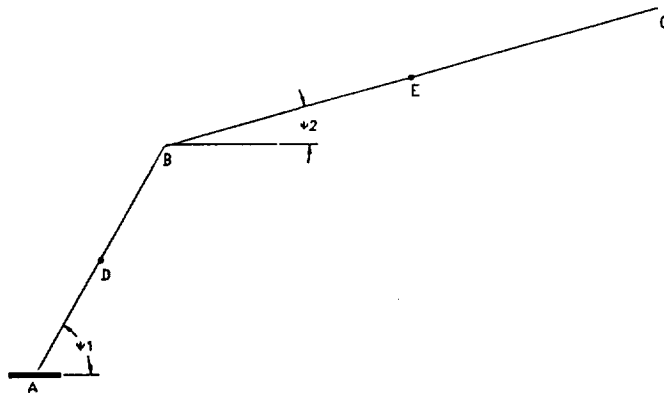


Fig.1 The Manipulator System Studied in the Paper

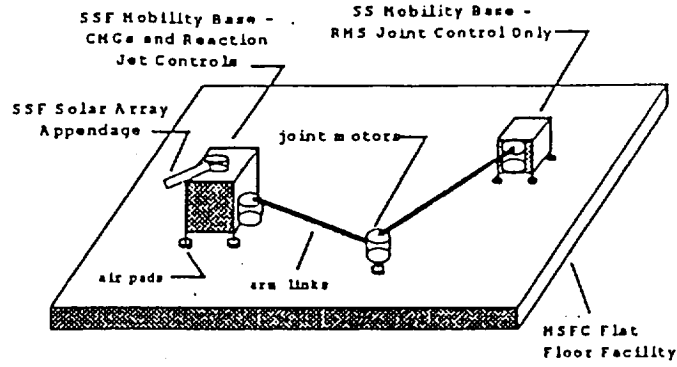


Fig.2 NASA MSFC Manipulator Testbed

The system studied consists of two flexible links and three revolute joints: shoulder joint A, elbow joint B and wrist joint C. In the paper, it is assumed that the base frame $X_0Y_0Z_0$ is fixed on the Shuttle assumed as a rigid body, with X_0 -axis along the joint A axis. The orientation of the Y_0 and Z_0 axes about the joint axis X_0 is chosen such that the resultant base frame forms a right-hand coordinate system. The frame $x_1y_1z_1$ for link 1 is defined as follows: the x_1 -axis coincides with the joint axis of the joint A, the z_1 -axis is along the axis of link 1. The frame $x_2y_2z_2$ is fixed at the joint B with x_2 -axis coinciding with the joint B axis, rotating with link 2, and z_2 -axis being along the axis of the link 2. Since the dimension of the end-effector can not be in general comparable with the dimensions of the two links, the end-effector will be abstracted as a rigid body represented by a mass point as a whole at the joint C.

Each joint of the manipulator arms is driven by an individual actuator. The control moments τ_A , τ_B , and τ_C are also acting on the revolute joints A, B, and C, respectively. The joint compliance is characterized by its torsional stiffness coefficient. The corresponding input joint torques are transmitted through the arm linkage to the end-effector, where the resultant force and moment act upon the environment. The configuration of the corresponding rigid-body system of the one with flexibility can be specified by the two joint angles ψ_1 and ψ_2 as shown in Figure 1.

3. KINEMATIC ANALYSIS UNDER RIGID-BODY LINK ASSUMPTION

The common method for kinematic analysis under rigid-body link assumption in the robotics society is based on the solution of Lagrangian equation,

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = Q_i \quad (3.1)$$

where, Lagrangian function L is defined as $L=T-U$, T and U are the kinetic and potential energies of the system, respectively. Q_i is the generalized force corresponding to the generalized coordinate q_i . For the two-arm system discussed in this paper, two rigid-body d.o.f.'s are chosen as the two angles ψ_1 and ψ_2 as shown in Figure 1, where points D and E are the mass centers of the two arms. The kinetic and potential energies for both arms can be expressed as follows, for arm 1:

$$T_1 = \frac{1}{2} m_1 v_D^2 + \frac{1}{2} I_D \dot{\psi}_1^2 = \frac{1}{6} m_1 L_1^2 \dot{\psi}_1^2, \quad U_1 = \frac{1}{2} m_1 g L_1 \sin \psi_1 \quad (3.2)$$

For arm 2:

$$T_2 = \frac{1}{2} m_2 v_E^2 + \frac{1}{2} I_E \dot{\psi}_2^2 = \frac{1}{2} m_2 [L_1^2 \dot{\psi}_1^2 + L_1 L_2 \dot{\psi}_1 \dot{\psi}_2 \cos(\psi_1 - \psi_2) + \frac{1}{3} L_2^2 \dot{\psi}_2^2]$$

$$U_2 = \frac{1}{2} m_2 g (2L_1 \sin \psi_1 + L_2 \sin \psi_2) \quad (3.3)$$

where, m_i , L_i are mass and length of the i th arm ($i=1$ or 2). The Lagrangian function is then,

$$L = T_1 + T_2 - U_1 - U_2$$

$$= \frac{1}{6} m_1 L_1^2 \dot{\psi}_1^2 + \frac{1}{2} m_2 [L_1^2 \dot{\psi}_1^2 + L_1 L_2 \dot{\psi}_1 \dot{\psi}_2 \cos(\psi_1 - \psi_2) + \frac{1}{3} L_2^2 \dot{\psi}_2^2]$$

$$- \frac{1}{2} g [(m_1 + 2m_2) L_1 \sin \psi_1 + m_2 L_2 \sin \psi_2] \quad (3.4)$$

Substituting Lagrangian function L into Eq.3.1 and taking corresponding derivatives, we derive the two Lagrangian equations:

$$\left(\frac{1}{3} m_1 + m_2 \right) L_1^2 \ddot{\psi}_1 + \frac{1}{2} m_2 L_1 L_2 \ddot{\psi}_2 \cos(\psi_1 - \psi_2) + \frac{1}{2} m_2 L_1 L_2 \dot{\psi}_2^2 \sin(\psi_1 - \psi_2)$$

$$+ K_A \psi_1 - K_B \psi_2 + \frac{1}{2} (m_1 + 2m_2) g L_1 \cos \psi_1 = \tau_A - \tau_B \quad (3.5-1)$$

$$\frac{1}{2} m_2 L_1 L_2 \ddot{\psi}_1 \cos(\psi_1 - \psi_2) + \frac{1}{3} m_2 L_2^2 \ddot{\psi}_2 - \frac{1}{2} m_2 L_1 L_2 \dot{\psi}_1^2 \sin(\psi_1 - \psi_2)$$

$$+ K_B \psi_2 + \frac{1}{2} m_2 g L_2 \cos \psi_2 = \tau_B - \tau_C \quad (3.5-2)$$

Isolating $\ddot{\psi}_1$ and $\ddot{\psi}_2$ in each of the above two equations, Eq.3.5 can be reorganized as,

$$\left[\frac{1}{3} m_1 + m_2 - \frac{3}{4} m_2 \cos^2(\psi_1 - \psi_2) \right] L_1^2 \ddot{\psi}_1 + \frac{3}{8} m_2 L_1^2 \dot{\psi}_1^2 \sin 2(\psi_1 - \psi_2) + \frac{1}{2} m_2 L_1 L_2 \dot{\psi}_2^2 \sin(\psi_1 - \psi_2)$$

$$+ K_A \psi_1 - \left[1 + \frac{3L_1 \cos(\psi_1 - \psi_2)}{2L_2} \right] K_B \psi_2 + \left(\frac{1}{2} m_1 + m_2 \right) g L_1 \cos \psi_1 - \frac{3}{4} m_2 g L_1 \cos \psi_2 \cos(\psi_1 - \psi_2)$$

$$= (\tau_A - \tau_B) - \frac{3L_1 (\tau_B - \tau_C) \cos(\psi_1 - \psi_2)}{2L_2} \quad (3.6-1)$$

$$\left[\frac{1}{2} m_2 \cos(\psi_1 - \psi_2) - \frac{2(\frac{1}{3} m_1 + m_2)}{3 \cos(\psi_1 - \psi_2)} \right] L_1 L_2 \ddot{\psi}_2 + \left(\frac{1}{3} m_1 + m_2 \right) L_1^2 \dot{\psi}_1^2 \tan(\psi_1 - \psi_2) + \frac{1}{2} m_2 L_1 L_2 \dot{\psi}_2^2 \sin(\psi_1 - \psi_2)$$

$$+ K_A \psi_1 - \left[1 + \frac{2(\frac{1}{3} m_1 + m_2) L_1}{m_2 L_2 \cos(\psi_1 - \psi_2)} \right] K_B \psi_2 + \frac{1}{2} (m_1 + 2m_2) g L_1 \cos \psi_1 - \frac{(\frac{1}{3} m_1 + m_2) g L_1 \cos \psi_2}{\cos(\psi_1 - \psi_2)}$$

$$= (\tau_A - \tau_B) - \frac{2(\frac{1}{3} m_1 + m_2) L_1 (\tau_B - \tau_C)}{m_2 L_2 \cos(\psi_1 - \psi_2)} \quad (3.6-2)$$

A set of two highly-coupled and non-linear simultaneous partial differential equations in Eq.3.6 must be solved numerically. The well-known Runge-Kutta method is used. To do so, we define

$$x_1 = \psi_1(t), \quad x_2 = \dot{\psi}_1(t), \quad x_3 = \psi_2(t), \quad x_4 = \dot{\psi}_2(t) \quad (3.7)$$

then, a set of four first-order simultaneous equations is generated,

$$\begin{aligned} \dot{x}_1 &= f_1(\cdot) = x_2 \\ \dot{x}_2 &= f_2(\cdot) = A_1 x_2^2 \sin 2(x_1 - x_3) + B_1 x_4^2 \sin(x_1 - x_3) + C_1 x_1 + D_1 x_3 + E_1 \cos x_1 \\ &\quad + F_1 \cos x_3 \cos(x_1 - x_3) + G_1 \cos(x_1 - x_3) + H_1 \\ \dot{x}_3 &= f_3(\cdot) = x_4 \end{aligned} \quad (3.8)$$

$$\begin{aligned} \dot{x}_4 &= f_4(\cdot) = A_2 x_2^2 \tan(x_1 - x_3) + B_2 x_4^2 \sin(x_1 - x_3) + C_2 x_1 + D_2 x_3 + E_2 \cos x_1 \\ &\quad + F_2 \frac{\cos x_3}{\cos(x_1 - x_3)} + G_2 \frac{1}{\cos(x_1 - x_3)} + H_2 \end{aligned}$$

where,

$$\begin{aligned} A_1 &= -\frac{\frac{3}{8} m_2 L_1^2}{\Delta_1}, \quad B_1 = -\frac{\frac{1}{2} m_2 L_1 L_2}{\Delta_1}, \quad C_1 = -\frac{K_A}{\Delta_1}, \quad D_1 = \frac{K_B}{\Delta_1} \left[1 + \frac{3 L_1 \cos(x_1 - x_3)}{2 L_2} \right] \\ E_1 &= -\frac{\frac{1}{2} (m_1 + 2 m_2) g L_1}{\Delta_1}, \quad F_1 = \frac{\frac{3}{4} m_2 g L_1}{\Delta_1}, \quad G_1 = -\frac{3 L_1 (\tau_B - \tau_C)}{2 L_2 \Delta_1}, \quad H_1 = \frac{\tau_A - \tau_B}{\Delta_1} \\ \Delta_1 &= \left[\frac{1}{3} m_1 + m_2 - \frac{3}{4} m_2 \cos^2(x_1 - x_3) \right] L_1^2 \end{aligned}$$

and

$$\begin{aligned} A_2 &= -\frac{(\frac{1}{3} m_1 + m_2) L_1^2}{\Delta_2}, \quad B_2 = -\frac{m_2 L_1 L_2}{2 \Delta_2}, \quad C_2 = -\frac{K_A}{\Delta_2}, \quad D_2 = \frac{K_B}{\Delta_2} \left[1 + \frac{2(\frac{1}{3} m_1 + m_2) L_1}{m_2 L_2} \right] \\ E_2 &= -\frac{(m_1 + 2 m_2) g L_1}{2 \Delta_2}, \quad F_2 = \frac{(\frac{1}{3} m_1 + m_2) g L_1}{\Delta_2}, \quad G_2 = -\frac{2(\frac{1}{3} m_1 + m_2) L_1 (\tau_B - \tau_C)}{m_2 L_2 \Delta_2}, \quad H_2 = \frac{\tau_A - \tau_B}{\Delta_2} \\ \Delta_2 &= \left[\frac{1}{2} m_2 \cos(x_1 - x_3) - \frac{2(\frac{1}{3} m_1 + m_2)}{3 \cos(x_1 - x_3)} \right] L_1 L_2 \end{aligned}$$

The iteration formula of the fourth-order Runge-Kutta method is as follows_[14]:

$$x_{i,j+1} = x_{i,j} + \frac{h}{6} (k_{i,1} + 2k_{i,2} + 2k_{i,3} + k_{i,4}) \quad i = 1, 2, 3 \text{ and } 4 \quad (3.9)$$

where

$$k_{i,1} = f_i(x_{1,j}; x_{2,j}; x_{3,j}; x_{4,j}; t)$$

$$\begin{aligned}
k_{i,2} &= f_i(x_{1,j} + \frac{h}{2}k_{1,1}; x_{2,j} + \frac{h}{2}k_{2,1}; x_{3,j} + \frac{h}{2}k_{3,1}; x_{4,j} + \frac{h}{2}k_{4,1}; t + \frac{h}{2}) \\
k_{i,3} &= f_i(x_{1,j} + \frac{h}{2}k_{1,2}; x_{2,j} + \frac{h}{2}k_{2,2}; x_{3,j} + \frac{h}{2}k_{3,2}; x_{4,j} + \frac{h}{2}k_{4,2}; t + \frac{h}{2}) \\
k_{i,4} &= f_i(x_{1,j} + hk_{1,3}; x_{2,j} + hk_{2,3}; x_{3,j} + hk_{3,3}; x_{4,j} + hk_{4,3}; t + h)
\end{aligned}$$

for the j th iteration, and h is the time interval. The accuracy of the method is in the order of h^5 . The motion of the rigid-body manipulator system can now be solved by using the iteration formula, Eq.3.9.

4. FLEXIBILITY ANALYSIS USING TRANSFER MATRIX METHOD

In this paper, the inclusion of the flexibility of the manipulator arms is treated by the distributed parameter approach along with the transfer matrix method. The function of the transfer matrix is to relate the linear and angular deflections, forces and moments at one point in a structure to those at another point. The derivation of the transfer matrices for bending and elongation of a beam element is shown in Refs.15 and 16, but is summarized here. The two flexible manipulator arms are represented by the Bernoulli-Euler equation and wave equation for their bending and elongation characteristics, respectively. The Bernoulli-Euler equation is in the form of

$$\frac{\partial^4 u_y}{\partial z^4} + \frac{1}{a_y^2} \frac{\partial^2 u_y}{\partial t^2} = 0 \quad (4.1)$$

where, $a_y^2 = k_y/m$, and $k_y = EI$ is the bending stiffness and m is the mass per length of the beam. The elongation vibration is described by the wave equation

$$\frac{\partial^2 u_z}{\partial z^2} - \frac{1}{a_z^2} \frac{\partial^2 u_z}{\partial t^2} = 0 \quad (4.2)$$

where, $a_z^2 = k_z/m$, and $k_z = EA$ is the axial stiffness. After separation of variables, Eqs.4.1 and 4.2 can be expressed in the spacial domain as

$$U_y'''(z) - \beta_y^4 U_y(z) = 0 \quad (4.3)$$

$$U_z''(z) + \beta_z^2 U_z(z) = 0 \quad (4.4)$$

where, $\beta_y^4 = \omega^2/a_y^2$, $\beta_z = \omega/a_z$, and ω is the circular natural frequency. The solutions to Eqs.4.3 and 4.4 are

$$U_y(z) = A \sin \beta_y z + B \cos \beta_y z + C \sinh \beta_y z + D \cosh \beta_y z \quad (4.5)$$

and

$$U_z(z) = M \sin \beta_z z + N \cos \beta_z z \quad (4.6)$$

where, A, B, C, D, M, N are the modal participation coefficients. If the state vector, $\{\theta\}$, is defined, then

$$\{\theta\} = [U_y, U_z, \psi_x, F_y, F_z, M_x]^T \quad (4.7)$$

where, U_y and U_z are the displacements along y- and z-axes, ψ_x is the rotary angle of the beam sections about x-axis, F_y and F_z are the shear and tensile forces respectively, M_x is the bending moment about x-axis. The state vectors at the two ends of a beam element are related by a matrix $[\Phi]$, that is,

$$\{\theta(L)\} = [\Phi]\{\theta(0)\} \quad (4.8)$$

where, the transfer matrix $[\Phi]$ involving bending and elongation of a beam element as described in Ref.16 is given in Eq.4.9,

$$[\Phi] = \begin{bmatrix} \frac{1}{2}(\cos\beta_y L + \cosh\beta_y L) & 0 & \frac{1}{2\beta_y}(\sin\beta_y L + \sinh\beta_y L) & \frac{1}{2k_y\beta_y^3}(-\sin\beta_y L + \sinh\beta_y L) & 0 & \frac{1}{2k_y\beta_y^2}(-\cos\beta_y L + \cosh\beta_y L) \\ 0 & \cos\beta_z L & 0 & 0 & \frac{1}{k_z\beta_z}\sin\beta_z L & 0 \\ \frac{1}{2}\beta_y(-\sin\beta_y L + \sinh\beta_y L) & 0 & \frac{1}{2}(\cos\beta_y L + \cosh\beta_y L) & \frac{1}{2k_y\beta_y^2}(-\cos\beta_y L + \cosh\beta_y L) & 0 & \frac{1}{2k_y\beta_y}(\sin\beta_y L + \sinh\beta_y L) \\ \frac{1}{2}k_y\beta_y^3(\sin\beta_y L + \sinh\beta_y L) & 0 & \frac{1}{2}k_y\beta_y^2(-\cos\beta_y L + \cosh\beta_y L) & \frac{1}{2}(\cos\beta_y L + \cosh\beta_y L) & 0 & \frac{1}{2}\beta_y(-\sin\beta_y L + \sinh\beta_y L) \\ 0 & -k_z\beta_z\sin\beta_z L & 0 & 0 & \cos\beta_z L & 0 \\ \frac{1}{2}k_y\beta_y^2(-\cos\beta_y L + \cosh\beta_y L) & 0 & \frac{1}{2}k_y\beta_y(-\sin\beta_y L + \sinh\beta_y L) & \frac{1}{2\beta_y}(\sin\beta_y L + \sinh\beta_y L) & 0 & \frac{1}{2}(\cos\beta_y L + \cosh\beta_y L) \end{bmatrix} \quad (4.9)$$

in which the elements at the 2nd and 4th rows and columns reflect the elongation, the others for the bending. Because the two arms are not in the same orientation, it is necessary to account for the alignment of the two arms. The relationship of the two local coordinate systems can be described by a coordinate transformation matrix $[T_{21}]$, that is,

$$\begin{Bmatrix} y_2 \\ z_2 \end{Bmatrix} = [T_{21}] \begin{Bmatrix} y_1 \\ z_1 \end{Bmatrix} \quad (4.10)$$

where,

$$[T_{21}] = \begin{bmatrix} \cos(y_2, y_1) & \cos(y_2, z_1) \\ \cos(z_2, y_1) & \cos(z_2, z_1) \end{bmatrix}$$

The transfer matrix of a revolute joint R has been derived in Ref.11. A revolute joint R is abstracted as a massless torsional spring with spring constant k_{ψ_R} through which two elements "a" and "b" are connected. The actuator fixed at the joint R will produce a control moment τ_R . The state vector $\{\theta\}_a$ at the end of element "a" is related to the state vector $\{\theta\}_b$ at the end of element "b" through the transfer matrix $[\Phi]$ of the joint R by

$$\{\theta\}_a = [\Phi_R]\{\theta\}_b + \{B_R\}\tau_R \quad (4.11)$$

where, the transfer matrix $[\Phi_R]$ of the joint R is given by

$$[\Phi_R] = \begin{bmatrix} [T_{ab}] & & \\ & 1 & \\ & & [T_{ab}] \\ & & & 1 \end{bmatrix} \quad (4.12)$$

and the control-influence vector,

$$\{B_R\} = \begin{bmatrix} 0 & 0 & \frac{1}{I_R s^2 + k_{\psi_R}} & 0 & 0 & -1 \end{bmatrix}^T \quad (4.13)$$

Applying the general expression, Eq.4.11, to the joints A and B, we find out that,

$$\{\theta(z_1 = 0)\}_0 = [\Phi_A]\{\theta(z_1 = 0)\}_1 + \{B_A\}\tau_A \quad (4.14)$$

and

$$\{\theta(z_1 = L_1)\}_1 = [\Phi_B]\{\theta(z_2 = 0)\}_2 + \{B_B\}\tau_B \quad (4.15)$$

where, the subscripts 0, 1 and 2 stand for the Shuttle Base, arm 1 and arm 2, respectively.

5. SYSTEM DYNAMIC EQUATIONS FOR A SPECIFIC CONFIGURATION

For a specific configuration at an arbitrary time instant, the system dynamic equation was derived in detail in Ref.11, that is

$$\{\theta(z_2 = L_2)\}_2 = [A]\{\theta(z_1 = 0)\}_0 + [B]\{\tau\} \quad (5.1)$$

where, the system matrix $[A] = [\Phi_2][\Phi_B]^{-1}[\Phi_1][\Phi_A]^{-1}$; the control-influence matrix $[B] = [-[\Phi_2][\Phi_B]^{-1}[\Phi_1][\Phi_A]^{-1}\{B_A\}, -[\Phi_2][\Phi_B]^{-1}\{B_B\}, -\{B_C\}]$; the control vector $\{\tau\} = [\tau_A, \tau_B, \tau_C]^T$. The element transfer matrices $[\Phi_\bullet]$'s and control influence vectors $\{B_\bullet\}$'s are associated with the two links and the three joints, respectively, recognized by the corresponding subscripts.

The dynamic properties at a certain configuration without control actions are the inherent properties of the system, which are varied with the change in configuration when the manipulator

system is in motion. The method to derive these inherent dynamic properties is straightforward. Consider the boundary conditions (B.C.'s) of the system in Figure 1, at the fixed end (attachment point to the Shuttle Base),

$$U_{y_0}(z_1 = 0) = U_{z_0}(z_1 = 0) = \Psi_{x_0}(z_1 = 0) = 0 \quad (5.2)$$

at the free end (end-effector),

$$F_{y_2}(z_2 = L_2) = F_{z_2}(z_2 = L_2) = M_{x_2}(z_2 = L_2) = 0 \quad (5.3)$$

Applying the B.C.'s, Eqs.5.2 and 5.3, to Eq.5.1 without control action, two equations can be derived,

$$[A_{12}] \begin{Bmatrix} F_{y_1}(z_1 = 0) \\ F_{z_1}(z_1 = 0) \\ M_{x_1}(z_1 = 0) \end{Bmatrix}_0 = \begin{Bmatrix} U_{y_2}(z_2 = L_2) \\ U_{z_2}(z_2 = L_2) \\ \Psi_{x_2}(z_2 = L_2) \end{Bmatrix}_2 \quad (5.4)$$

and

$$[A_{22}] \begin{Bmatrix} F_{y_1}(z_1 = 0) \\ F_{z_1}(z_1 = 0) \\ M_{x_1}(z_1 = 0) \end{Bmatrix}_0 = 0 \quad (5.5)$$

where, $[A_{12}]$ and $[A_{22}]$ are the block matrices of the matrix $[A]$. The condition for Eq.5.5 having non-trivial solution is that

$$DET[A_{22}] = 0 \quad (5.6)$$

Eq.5.6 is the characteristic equation of the system from which the natural frequencies ω 's can be derived. After obtaining the natural frequencies from Eq.5.6, we can derive the mode shape functions from the equation below,

$$[G]\{\zeta\} = 0 \quad (5.7)$$

The detailed derivation of Eq.5.7 can be found in Ref. 16. The vector $\{\zeta\}$ consists of the modal participation coefficients for the beams 1 and 2 appearing in Eqs.4.5 and 4.6, that is, $\{\zeta\} = [A_1, B_1, C_1, D_1, M_1, N_1, A_2, B_2, C_2, D_2, M_2, N_2]^T$. Normalizing Eq.5.7 with $N_2=1$, Eq.5.7 can be solved to obtain the modal participation coefficients for the two beams, thereby the mode shape functions can be obtained based on the solution equations, that is, Eqs.4.5 and 4.6, for the i th beam,

$$U_{y_i}(z_i) = A_i \sin \beta_{y_i} z_i + B_i \cos \beta_{y_i} z_i + C_i \sinh \beta_{y_i} z_i + D_i \cosh \beta_{y_i} z_i \quad (5.8)$$

and

$$U_{z_i}(z_i) = M_i \sin \beta_{z_i} z_i + N_i \cos \beta_{z_i} z_i \quad (5.9)$$

It is assumed that the control actions are related to the feedbacks of the nodal displacements and velocities, that is,

$$\tau_A = [k_A] \{ \theta(z_1 = 0) \}_1 = [k_{A_1} + k_{A_4}S, k_{A_2} + k_{A_5}S, k_{A_3} + k_{A_6}S, 0, 0, 0] \{ \theta(z_1 = 0) \}_1 \quad (5.10)$$

$$\tau_B = [k_B] \{ \theta(z_2 = 0) \}_2 = [k_{B_1} + k_{B_4}S, k_{B_2} + k_{B_5}S, k_{B_3} + k_{B_6}S, 0, 0, 0] \{ \theta(z_2 = 0) \}_2 \quad (5.11)$$

$$\tau_C = [k_C] \{ \theta(z_2 = L_2) \}_2 = [k_{C_1} + k_{C_4}S, k_{C_2} + k_{C_5}S, k_{C_3} + k_{C_6}S, 0, 0, 0] \{ \theta(z_2 = L_2) \}_2 \quad (5.12)$$

Inserting the control actions into Eq.5.1 and combining the similar terms, the closed-loop system dynamic equation follows,

$$\{ \theta(z_2 = L_2) \}_2 = [\bar{A}] \{ \theta(z_1 = 0) \}_0 \quad (5.13)$$

where, $[\bar{A}] = [\bar{\Phi}_2][\bar{\Phi}_B]^{-1}[\bar{\Phi}_1][\bar{\Phi}_A]^{-1}$, and

$$[\bar{\Phi}_A] = [\Phi_A] + \{B_A\}[k_A], [\bar{\Phi}_B] = [\Phi_B] + \{B_B\}[k_B], [\bar{\Phi}_2] = ([I] + \{B_C\}[k_C])^{-1}[\Phi_2].$$

By applying the B.C.'s, Eqs.5.2 and 5.3, the closed-loop characteristic equation, $DET[\bar{A}_{22}] = 0$, can be derived, where, $[\bar{A}_{22}]$ is a block matrix of the matrix $[\bar{A}]$, from which the closed-loop poles can be found.

6. SUPERPOSING RIGID-BODY KINEMATICS AND FLEXIBILITY EFFECT

This paper conceives a new mathematical treatment for dynamic analysis of large flexible manipulator systems. The essence of the idea is to separate the kinematics and flexibility analyses as two independent but successive steps in a small time interval. Section 3 of this paper gave the kinematic analysis assuming a rigid-body link based on the Lagrangian equation method, using the Runge-Kutta numerical approach. Sections 4 and 5 provided a method for system dynamic analysis due to flexibility at a specific configuration of the manipulator system. Compared with the macroscopic motion of the manipulator system, the motion resulted from flexibility is only a "microcosmic" motion. Only after a long-term effect is accumulated will the flexibility effect be significant. It allows us, therefore, to make the assumptions: at a certain configuration of the manipulator system, the deflections and the rates of deflection of the arms due to flexibility are small, and the elongation deformations are high-order infinitesimal so that they are neglected. Superposing the rigid-body motion and the motion due to flexibility, we have

$$\bar{\psi}_1(z_1, t) = \psi_1(t) + d\psi_1(z_1, t) \quad \text{and} \quad \bar{\psi}_2(z_2, t) = \psi_2(t) + d\psi_2(z_2, t) \quad (6.1)$$

where, the small perturbation can be assumed as (cf. Fig.3),

$$d\psi_1(z_1, t) = \frac{1}{z_1} y_1(z_1, t) \quad \text{and} \quad d\psi_2(z_2, t) = \frac{1}{z_2} y_2(z_2, t) \quad (6.2)$$

Therefore, the two generalized coordinates defining the instantaneous motions of the two arms in the Lagrangian equation, Eq.3.5, would be

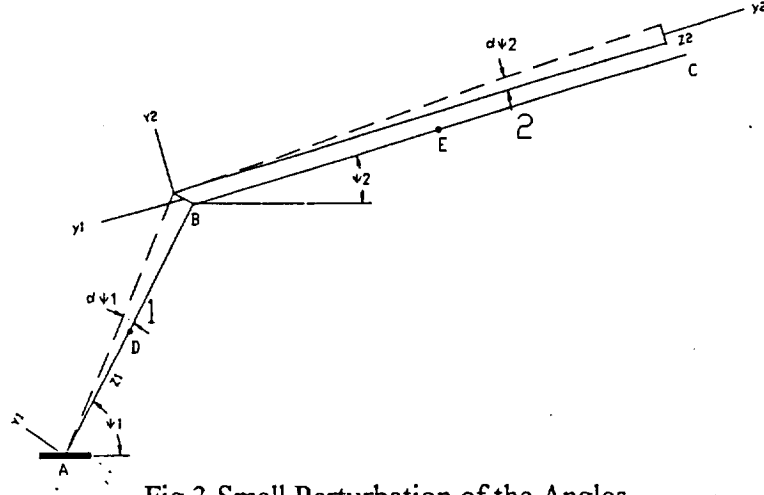


Fig.3 Small Perturbation of the Angles

$$\psi_1(t) = \bar{\psi}_1(z_1, t) - \frac{1}{z_1} y_1(z_1, t) \quad \text{and} \quad \psi_2(t) = \bar{\psi}_2(z_2, t) - \frac{1}{z_2} y_2(z_2, t) \quad (6.3)$$

Substituting Eq.6.3 into Eq.3.6 and neglecting high-order infinitesimals, we can express the Lagrangian equations in terms of $\bar{\psi}_1$ and $\bar{\psi}_2$ as follows,

$$\begin{aligned} \ddot{\bar{\psi}}_1 = & A_1 \dot{\bar{\psi}}_1^2 \sin 2(\bar{\psi}_1 - \bar{\psi}_2) + B_1 \dot{\bar{\psi}}_2^2 \sin(\bar{\psi}_1 - \bar{\psi}_2) + C_1 \bar{\psi}_1 + D_1 \bar{\psi}_2 + E_1 \cos \bar{\psi}_1 \\ & + F_1 \cos \bar{\psi}_2 \cos(\bar{\psi}_1 - \bar{\psi}_2) + G_1 \cos(\bar{\psi}_1 - \bar{\psi}_2) + H_1 + \frac{1}{z_1} \ddot{y}_1 \end{aligned} \quad (6.4-1)$$

$$\begin{aligned} \ddot{\bar{\psi}}_2 = & A_2 \dot{\bar{\psi}}_1^2 \tan(\bar{\psi}_1 - \bar{\psi}_2) + B_2 \dot{\bar{\psi}}_2^2 \sin(\bar{\psi}_1 - \bar{\psi}_2) + C_2 \bar{\psi}_1 + D_2 \bar{\psi}_2 + E_2 \cos \bar{\psi}_1 \\ & + F_2 \frac{\cos \bar{\psi}_2}{\cos(\bar{\psi}_1 - \bar{\psi}_2)} + G_2 \frac{1}{\cos(\bar{\psi}_1 - \bar{\psi}_2)} + H_2 + \frac{1}{z_2} \ddot{y}_2 \end{aligned} \quad (6.4-2)$$

Defining $w_1 = \bar{\psi}_1$, $w_2 = \dot{\bar{\psi}}_1$, $w_3 = \bar{\psi}_2$, and $w_4 = \dot{\bar{\psi}}_2$, a set of four first-order simultaneous equations is obtained which is similar to Eq.3.8,

$$\begin{aligned} \dot{w}_1 &= w_2 \\ \dot{w}_2 &= A_1 w_2^2 \sin 2(w_1 - w_3) + B_1 w_4^2 \sin(w_1 - w_3) + C_1 w_1 + D_1 w_3 + E_1 \cos w_1 \\ &+ F_1 \cos w_3 \cos(w_1 - w_3) + G_1 \cos(w_1 - w_3) + H_1 + \frac{1}{z_1} \ddot{y}_1 \\ \dot{w}_3 &= w_4 \\ \dot{w}_4 &= A_2 w_2^2 \tan(w_1 - w_3) + B_2 w_4^2 \sin(w_1 - w_3) + C_2 w_1 + D_2 w_3 + E_2 \cos w_1 \\ &+ F_2 \frac{\cos w_3}{\cos(w_1 - w_3)} + G_2 \frac{1}{\cos(w_1 - w_3)} + H_2 + \frac{1}{z_2} \ddot{y}_2 \end{aligned} \quad (6.5)$$

The only difference between Eq.6.5 and Eq.3.8 is the inclusion of \ddot{y}_1 and \ddot{y}_2 , which represents the effect of flexibility, and can be solved based on the formulation described in Sections 4 and 5 as long as the instantaneous configuration is specified and the motion at the end of previous time

interval is known. By using the Runge-Kutta procedure in Section 3, Eq.6.5 can be solved numerically. A solution is thus obtained which represents the superposition of rigid-body kinematics and flexibility effect.

7. SIMULATION EXAMPLES

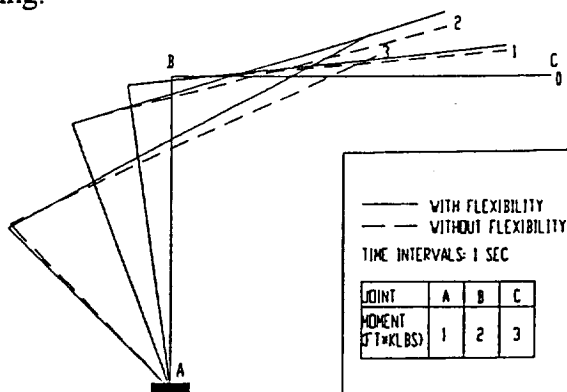
As mentioned earlier, the manipulator system studied here is a similitude of a NASA MSFC manipulator testbed for the research of the berthing operation of the Space Shuttle to the Space Station. The mechanical properties of the system are listed in Table 1.

Table 1 Mechanical Properties of the Two-arm Manipulator System

	Beam 1	Beam 2
Beam Length, L, (in.)	60.0	120.0
Sectional Area, A, (in. ²)	50.27	12.57
Second Moment of the Cross Section, I, (in. ⁴)	107.0	107.0
Modulus of Elasticity, E, (psi)	201.06	12.57
Mass per length, (slug/in.)	0.1526	0.0382

The initial conditions of the system are assumed at $\psi_1 = 90^\circ$, $\psi_2 = 0^\circ$, $\dot{\psi}_1 = 0$, and $\dot{\psi}_2 = 0$. First, the difference between the results of under rigid-body link assumption and with flexibility effects are noted by three examples as shown in Fig.4. These examples exhibit the motions of the manipulator system from the specified initial conditions under the actions of gravity force, inertia force, and the constant joint control moments τ_A , τ_B , and τ_C as shown in each figure. The solid lines represent the motions under rigid-body link assumption, while the dashed lines represent the motions with flexibility effects. For clarity, the vibratory wave shapes for flexible beams were neglected in the figures.

Next, several examples demonstrate the effectiveness for end-effector vibration suppression. The motion of the manipulator system is a continuous process, which stimulates vibration of the system at every time instant. The vibration suppression action continues throughout the whole process without interruption. The joint control moments will always change themselves based upon the control law given in Eqs.5.10 to 5.12. They play the roles of both actuating the manipulator system to fulfill a certain task and alleviating vibratory fluctuation while the system is operating.



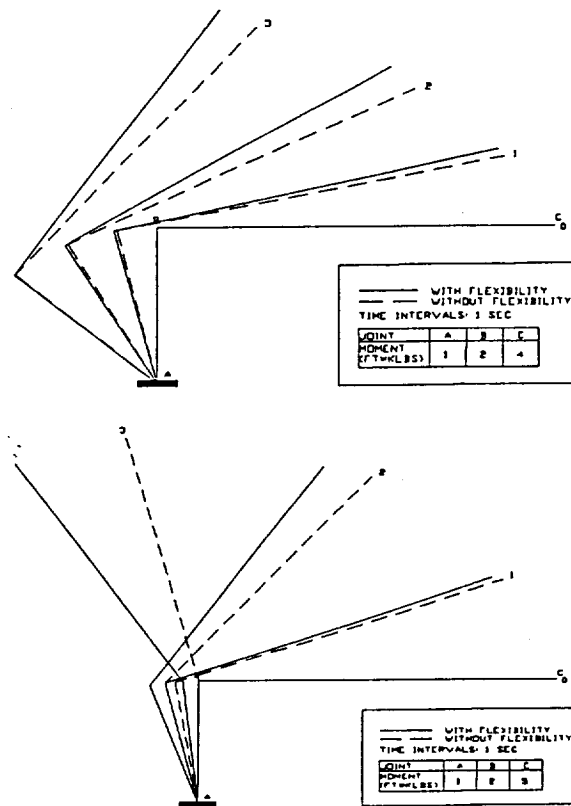


Fig.4 The Difference Between the Results Of Under Rigid-Body Link Assumption And With Flexibility Effects

Figs.5 to 7 demonstrate the effectiveness for the end-effector vibration suppression at instantaneous positions 1, 2, 3 as shown in Fig.4, with the initial joint control moments $\tau_A=1$ ft-klb, $\tau_B=2$ ft-klb, and $\tau_C=4$ ft-klb. Both time histories without control (left) and with control (right) are shown in the figures for comparison. The vibratory motion of the end-effector is described in the second link's coordinate system, $x_2y_2z_2$, as defined in Section 2. The upper two figures in Figs.5 to 7 represent the results in y_2 -direction, the lower two in z_2 -direction. The instantaneous vibration can be suppressed in about 0.3 second for all positions studied.

8. CONCLUDING REMARKS

A new mathematical treatment for dynamic analysis of large flexible manipulator systems is derived. An extremely complex analytical chore is resolved into two relatively simpler problems, the complexity of the dynamic analysis of large flexible manipulator systems is, therefore, mathematically simplified to a realistically acceptable extent for practical manual symbolic derivation of the equations of motion. Since the equation of motion is a set of highly-coupled and non-linear simultaneous partial differential equations, the Runge-Kutta numerical procedure has been used to solve the equations. As an example, the vibration suppression problem of a similitude of a NASA MSFC manipulator testbed has been investigated in the paper.

The computational results show that the proposed method is very effective for end-effector vibration suppression for a large flexible manipulator system.

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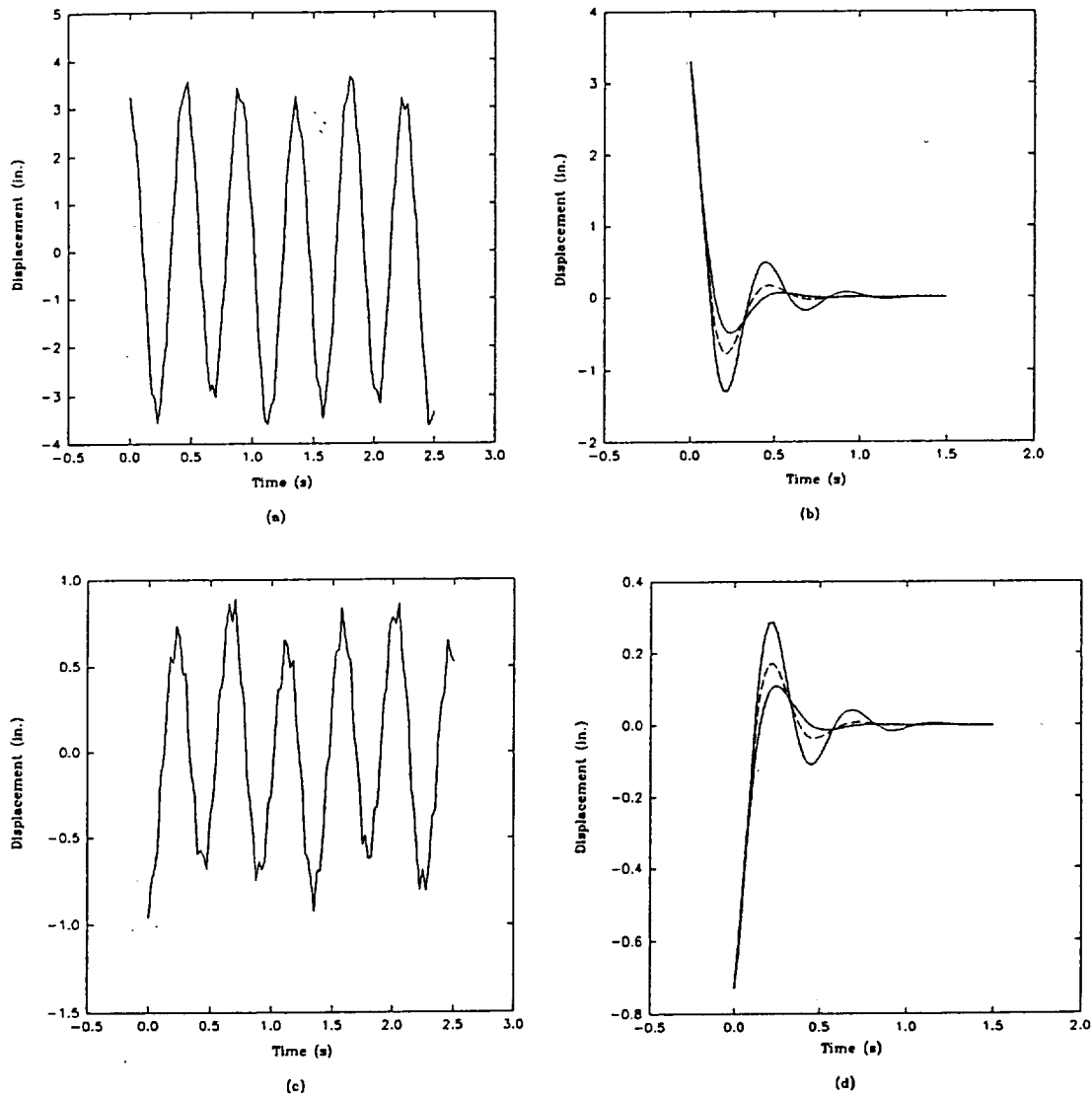


Fig.5 End-Effector Vibration Suppression at Instantaneous Position: $\psi_1 = 103.2^\circ$, $\psi_2 = 3.6^\circ$

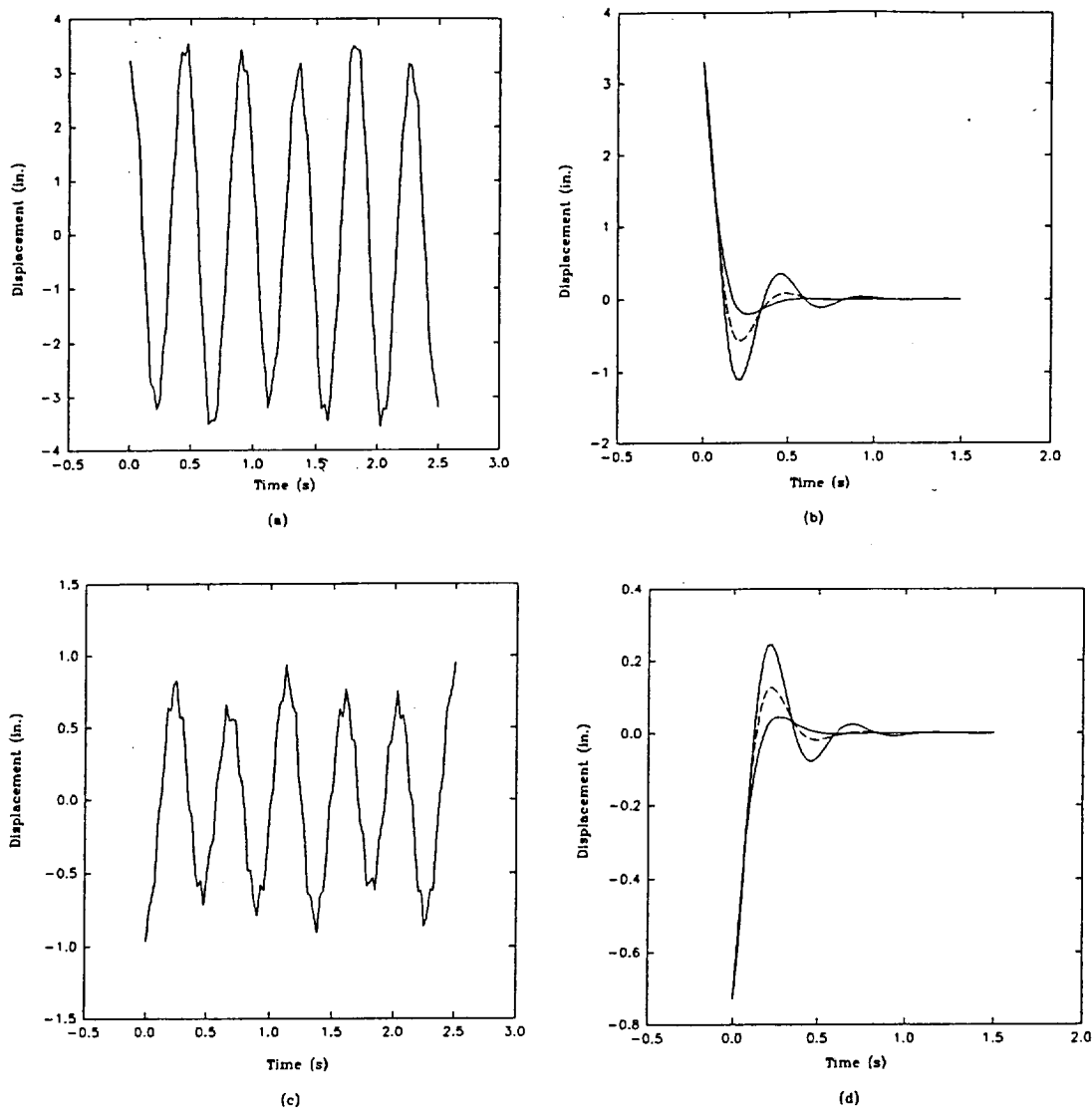


Fig.6 End-Effector Vibration Suppression at Instantaneous Position: $\psi_1 = 121.8^\circ$, $\psi_2 = 10.2^\circ$

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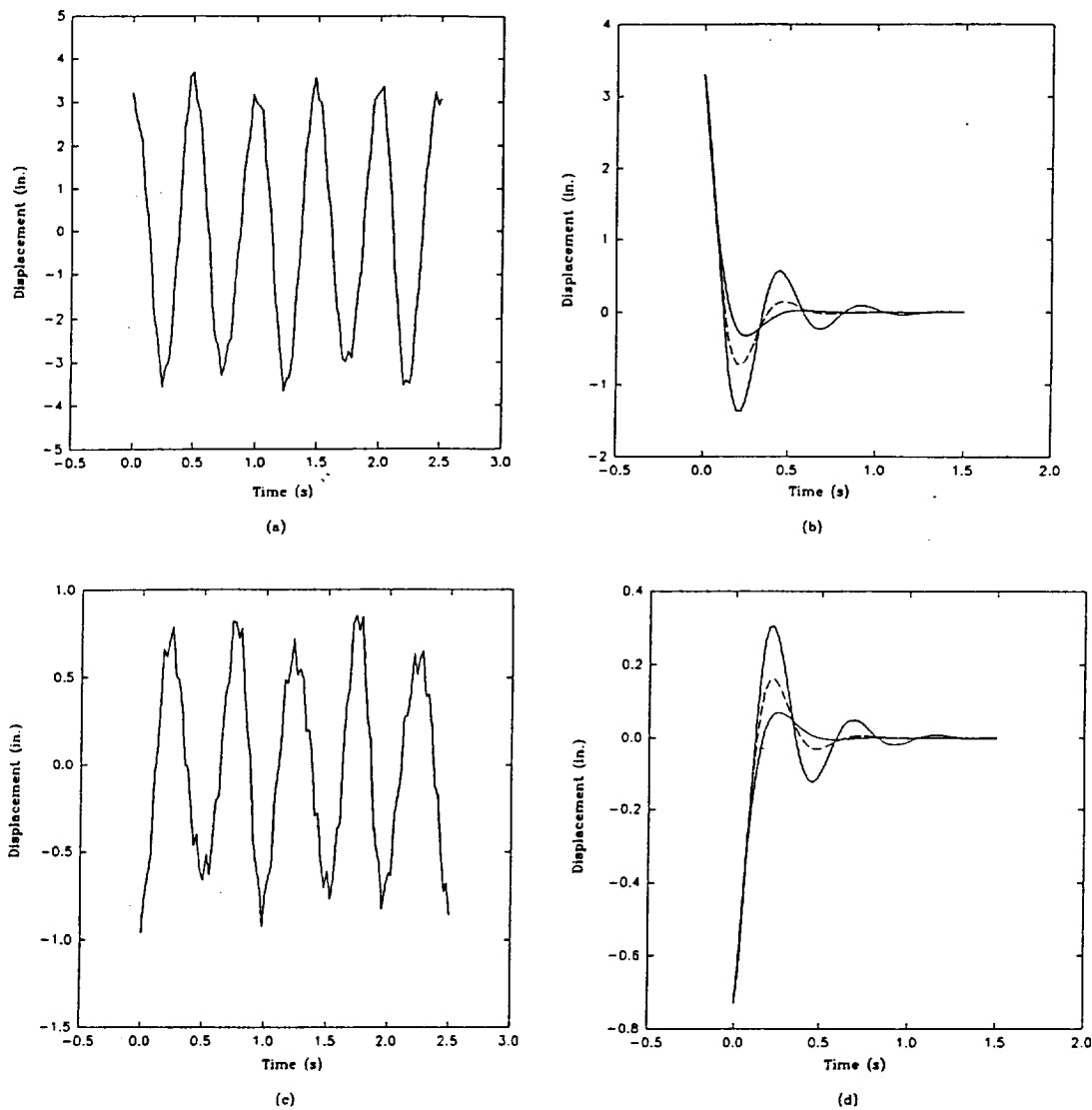


Fig.7 End-Effector Vibration Suppression at Instantaneous Position: $\psi_1 = 149.0^\circ$, $\psi_2 = 18.1^\circ$

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